# Towards Rotordynamic Analysis with COMSOL Multiphysics™

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**Abstract:** In this paper a pre-study on using COMSOL Multiphysics™ for rotordynamic analysis is presented. It is concluded that it is possible to use COMSOL Multiphysics™ to perform rotordynamical analysis. However, there are no standard environment for rotordynamics, hence the user has to extend the structural model with the rotordynamics effect such as gyroscopic effect and rotordynamical coefficients. By using a standard finite element code for rotordynamical analysis, one can take the benefit of using detailed model of structures and rotating component in rotordynamical analysis.

**Keywords:** Rotordynamics, fluid-rotor, electro-mechanics

## 1. Introduction

Rotordynamical calculations are normally carried out with special purpose software's based on Timoshenko beam elements. Forces and moments from interconnections such as bearings, seals, impellers, magnetic fields and the supporting structure are included as added mass, stiffness and damping coefficients. These coefficients are dependent on the operational speed and the rotor whirling frequencies. Rotors are also subjected to gyroscopic effect which is speed dependent. Hence, rotordynamical analysis has to be carried out for the whole speed range that the rotor will operate within; i.e. analysis of natural frequencies, damping, unbalance and other harmonic responses have to be carried out at several speeds to understand the dynamic behaviour of the rotor system.

From a practical perspective, it is easy to understand the use of special purpose rotordynamical codes. On the other side, the modelling of the machine has to be carried out twice, once for the rotordynamical performance and once for the structural analysis of the machine. Some structural analysis would also be improved if one combines them with rotordynamical analysis for example seismic and sea load analysis. Rotordynamical analysis would also be improved if one could do a better representation of structure.

## 2. Rotordynamic modelling

Traditionally, Euler-Bernoulli or Timoshenko beam elements are used in rotordynamical models. Blades, rotor poles etc are modelled as added mass and inertia. Lately, the interest of using solid elements for rotors and shell elements for blades has increased in the rotordynamical society. Rotordynamical problems are normally modelled as a non-rotating shaft, where effects due to rotation are added to the Equations of motion.

The Equation of motion for lateral vibration of a rotor is given by:

$$\bar{M}\ddot{\vec{x}} + \left(\bar{C} + \Omega\bar{\mathrm{G}}\right)\dot{\vec{x}} + \bar{K}\vec{x} = \vec{F}(t)$$

where $\bar{M}$ is the mass matrix, $\bar{C}$ is the damping matrix, $\bar{\mathrm{G}}$ is the gyroscopic matrix, $\bar{K}$ is the stiffness matrix and $\vec{F}(t)$ is the time dependent force and moment vector. The mass, stiffness and damping matrices include both the mechanical elements of the rotor system as well as the interactions at bearings, seals and magnetic fields.

Forces and moments due to bearings, seals, impellers and magnetic fields are normally included as stiffness, damping and mass coefficients, which are speed dependent (and in some cases also dependent on the whirling frequency), in a rotordynamical software. The coefficients are based on measurements, empirical formulas or numerical calculations. COMSOL Multiphysics™ has the capability to solve the different physical problems such as thin film flows, turbulent flows, magnetic fields, hence COMSOL Multiphysics™ has the capability to calculate the rotordynamical coefficients of multiphysical interactions as well as calculating the transient interaction between the rotor and the surrounded fluid or magnetic field.

An important issue is the supporting structure that has to be represented in the rotordynamical model. A general purpose finite element program, such as COMSOL Multiphysics™, has great possibilities to include a detailed model for the structure and foundation plate, which is a better representation than just using bearing foundation stiffness and mass that is quite common in rotordynamical software. Another benefit of using general purpose software is that one can use CAD-models to model the rotor, instead of filling in a table of the rotor dimensions.

Below are some descriptions of how to use COMSOL Multiphysics™ for rotordynamical calculations and what the user has to implement. In extension to standard rotordynamical analysis, special features as coupled rotor-fluid/magnetic field calculations as well as solid element rotordynamics are discussed.

### 2.1 Gyroscopic effect

For beam elements, gyroscopic effect can be added as an edge load according to:

$$M_L(x) = -0.5 * beam.rho * \pi * \left(sqrt(beam.area)\right)^4 * \Omega * thyt$$

$$M_L(y) = 0.5 * beam.rho * \pi * \left(sqrt(beam.area)\right)^4 * \Omega * thxt$$

where $\Omega$ is the rotor spin speed. Gyroscopic moments due to the discs can be added as point loads (which is the common way of modelling discs in rotordynamical software):

$$M_P(x) = -J_P * \Omega * thyt$$

$$M_P(y) = J_P * \Omega * thxt$$

Where $J_P$ is the disc polar moment of inertia.

### 2.2 Unbalance and harmonic responses

Unbalance and harmonic responses are the most common loads on a rotordynamical system. The unbalance or other harmonic load can easily be implemented as a point load in COMSOL Multiphysics™:

$$F_P(x) = m * e * \Omega\hat{}2 * \cos(\Omega * t)$$

$$F_P(y) = m * e * \Omega\hat{}2 * \sin(\Omega * t)$$

where m is the mass of the disc and e is the distance of the unbalance.

### 2.3 Structure representation

Combing different elements is easy in COMSOL Multiphysics™ by using an ”integrator” to couple plate, solid and beam elements. In a rotordynamic application one can already today use shell elements to model the machine foundation, solid elements to model the machine structure frame and connect them thru bearing with beam elements.

### 2.4 Rotordynamical coefficients

Rotordynamical coefficients due to multiphysical interactions can numerically be determined by different methods. The main idea is to perturb a rotor surrounded by a fluid or magnetic field and obtain the forces (and the moments) due to the perturbation. These forces can normally be written as a function of displacement, velocity and acceleration, hence one can derive stiffness, damping and mass coefficients for the interactions. These coefficients are modelled as spring, dampers or masses connected between the rotor and structure (or grounded foundation). An example of the implementation of bearing stiffness and damping in the x-direction is given by:

$$\begin{gathered} Fp(x) = \\ -((u - u2) * k_{xx} + (v - v2) * k_{xy} \\ +(beam.u_{tx} - beam2.u_{tx}) * c_{xx} \\ +(beam.u_{ty} - beam2.u\_ty) * c_{xy}) \end{gathered}$$

where $k_{xx}$ is the direct stiffness, $k_{xy}$ the cross-coupled damping, $c_{xx}$ is the direct stiffness and $c_{xy}$ the cross-coupled damping. Note that beam2 is the bearing support structure in this example.

For fluid-film bearings; coefficients are normally calculated by first obtaining the static equilibrium position of the rotor inside the lubrication and using a small perturbation around the equilibrium position [1]. The equilibrium position is dependent on the speed as well as the load, hence one need to calculate the coefficients for the whole speed and load range. One should also make calculations for minimum, maximum and nominal bearing preload. For tilting pad bearings, one should also include a structural model of the pads in order to obtain correct bearing coefficients.

Rotordynamical coefficients of seals, impellers and electromagnetic fields are normally determined with a prescribed whirling motion of the rotor and the coefficients are found by signal processing of the obtained forces and moments [2]. Asynchronous (i.e. when whirling frequency doesn't correspond to the rotational frequency) coefficients are found by variation of the whirling frequency. The latter requires several simulations, especially since the coefficients also can be dependent on speed and load.

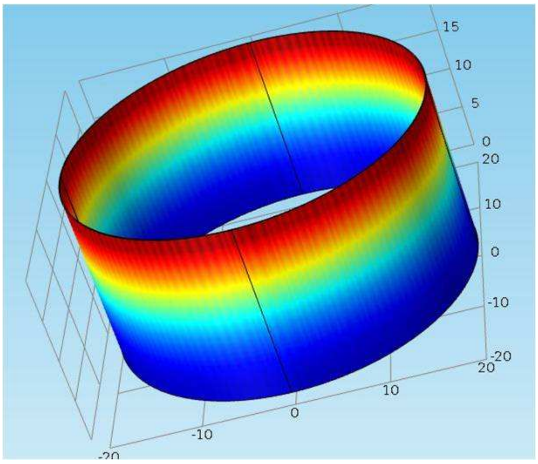


**Figure 1 Numerical determination of rotordynamical coefficients of a plain seal**

If the rotordynamical coefficients are dependent on whirling frequency, it is computational costly to calculate them since one need to carry out many simulations. For electromechanical applications, the Impulse method [3] has been introduced to determine the rotordynamical coefficients. The Impulse method only needs one simulation for each speed and loads to determine the coefficients. Instead of using many different whirling motions, it uses an impulse motion, which excites a broad number of frequencies (including backward frequencies).

Rolling element bearings can be implemented in a similar way, but uses structural elements instead of fluid or AC/DC elements.

In addition to the numerical identification mentioned above, empirical formulas for the different multiphysical interactions can easily be added as point loads.

### 2.5 Other interconnections

Couplings and gears are normally included as stiffness, inertia and gear ratio in rotordynamical codes. Identification of the stiffness and inertia can be done in a similar procedure as described above.

Control system can be added by adding the equations of the control system as a Global ODE. This can be used for active magnetic bearings, process control and variable speed drives.

## 3. Rotordynamic analysis

Since the properties of a rotating system is changing due to speed and process load, one need to analyse the system over the whole speed range. Below is a discussion of the most common analysis within rotordynamics. COMSOL Multiphysics™ has already good capability to do most of the post-processing described below.

### 3.1 Critical speed map

Critical speed map is carried out by variation of the total bearing stiffness (structure and fluid film stiffness) from a low value to a high value and solve the eigenvalue problem for the whole speed range. Critical speeds (speed at which an external forcing excite one of the system's eigenfrequencies) are plotted as a function of bearing stiffness. One can use the critical speed map to decide which bearing properties that one should have to obtain decent separation marginal to critical speeds, avoid structural vibration and obtain decent damping in the bearings. Additionally, the mode shape can be plotted for the critical speeds, which gives information of how the vibration shape will change with changed bearing stiffness.

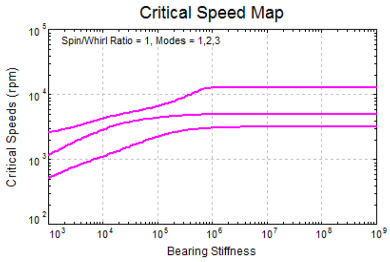


**Figure 2 Example of a criticial speed map**

### 3.2 Eigenfrequency and stability

A Campbell plot is used to show how the eigenfrequency is changed with respect to the speed. Normally the main excitation frequencies are added into a Campbell plot and the intersection with the eigenfrequency lines gives the critical speeds. When the mode shape is plotted, one does also need to have a notation if the mode is a forward whirling (the direction of the vibration is the same as the rotation) or a backward whirling (the direction of the vibration is opposite to the vibration) mode, it can be done with arrows or by adding a sign to the eigenferquency.

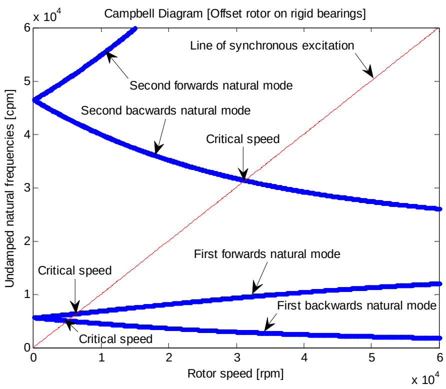


**Figure 3 Example of a Campbell plot**

Stability is normally defined as a required modal damping. The damping can be plotted as function of speed, or in a root locus plot where the eigenfrequency is plotted as a function of damping.

### 3.3 Unbalance and harmonic response

Unbalance response and harmonic responses are plotted for the node(s) with the unbalance/harmonic load. Amplitude in x- and y-directions as well as the phase between the excitation force and response is normally plotted. Operational deflected shapes at nominal speed(s) and passage of critical speeds are normally also presented.

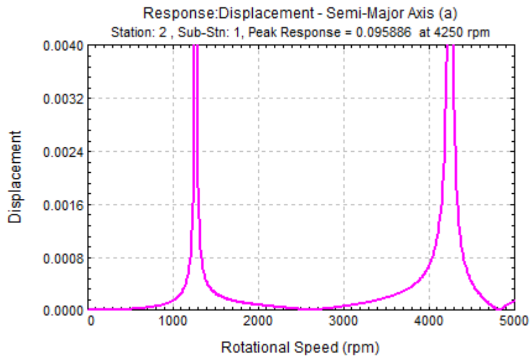


**Figure 4 Example of unbalance response**

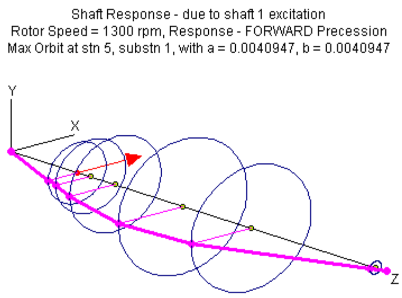


**Figure 5 Example of operating deflected shape**

## 7. Conclusions

It is concluded that it is possible to carry out rotordynamic modelling with COMSOL Multiphysics™, however the user needs to implement typical rotordynamic loads. Until now, the authors of this paper has used external software for post-processing the results, but it should be possible to do most post-processing in the COMSOL result node.

## 8. References

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