Optimal placement of piezoelectric plates to control multimode vibrations of rotating beam

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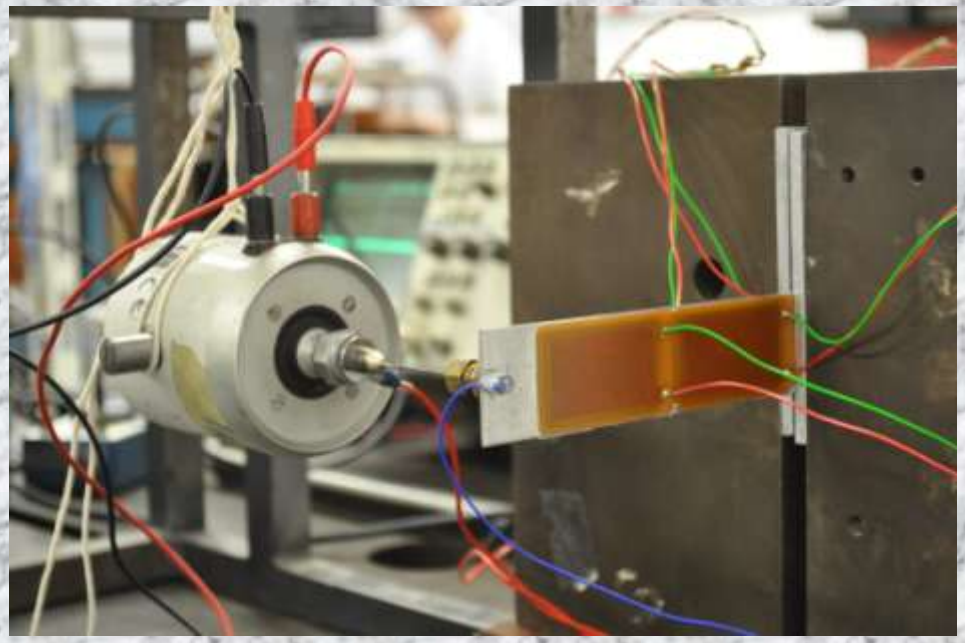
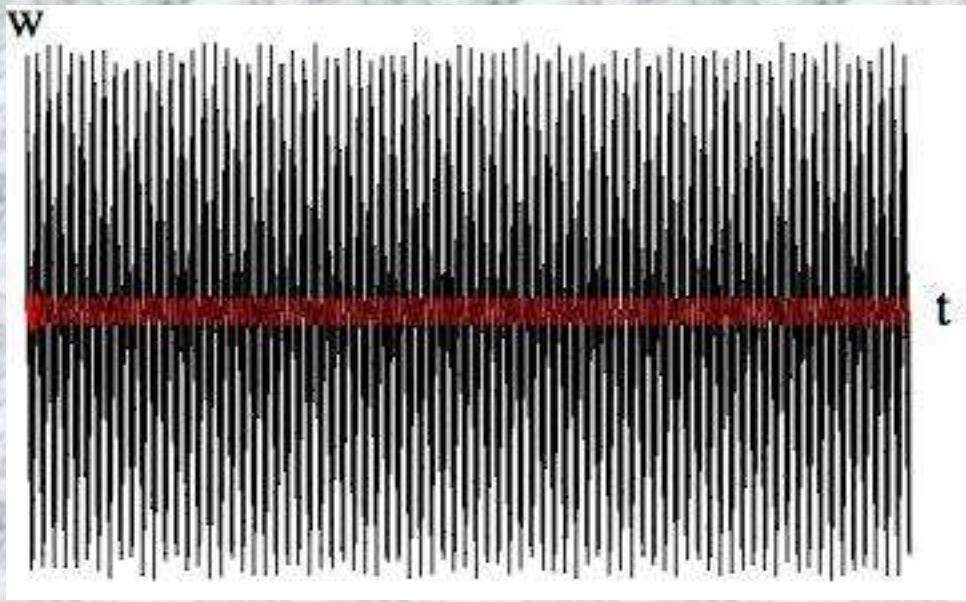
Gas Turbine Blade Vibrations

Fatigue Related Phenomena

Catastrophic Failures and Reduction of the Blades Life



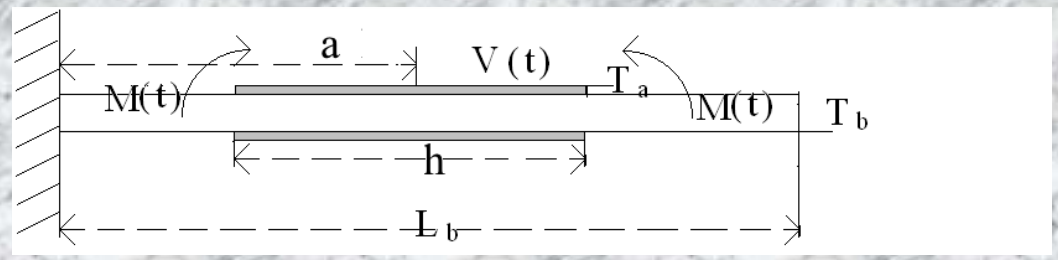


without control

with control

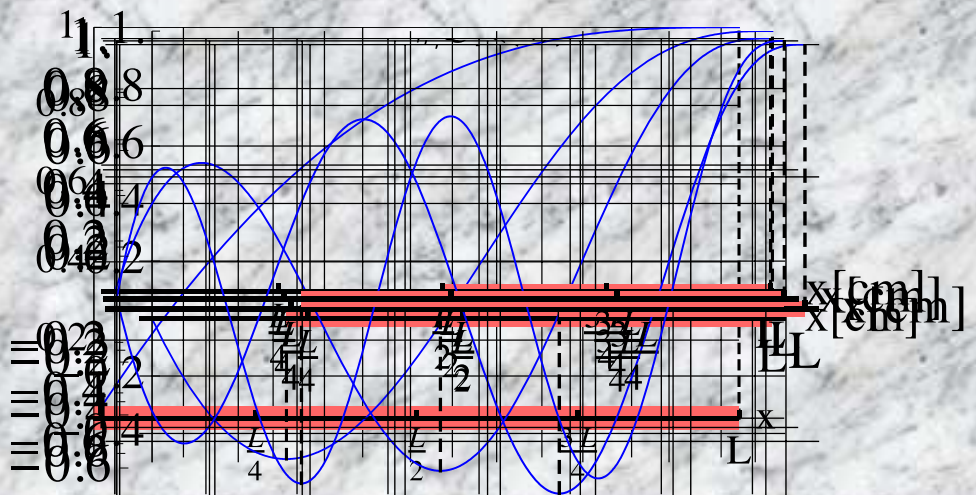
Optimal placement of piezoelectric plates: single mode

$$F(t)=F_0\cos(\omega_1 t)$$



Pin Force Model

$$dL^P = M_a\left[f_i'\left(a+\frac{h}{2}\right)-f_i'\left(a-\frac{h}{2}\right)\right]$$



Optimal placement of piezoelectric plates: multimode control

$$\begin{cases} \mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{B}(a,h)\sum_{k=1}^{N_s} V_h Cos(\omega_h t) \\ w(a,h,x,t) = \sum_{n=1}^{N} X_n(t)\phi_n(x) \end{cases}$$

piezoelectric effect

Ns=2 ⟶ 2 eigenmodes considered: i and j

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{B}(a,h)\left[V_i Cos(\omega_i t) + V_j Cos(\omega_j t)\right]$$

Optimal placement of piezoelectric plates: multimode control

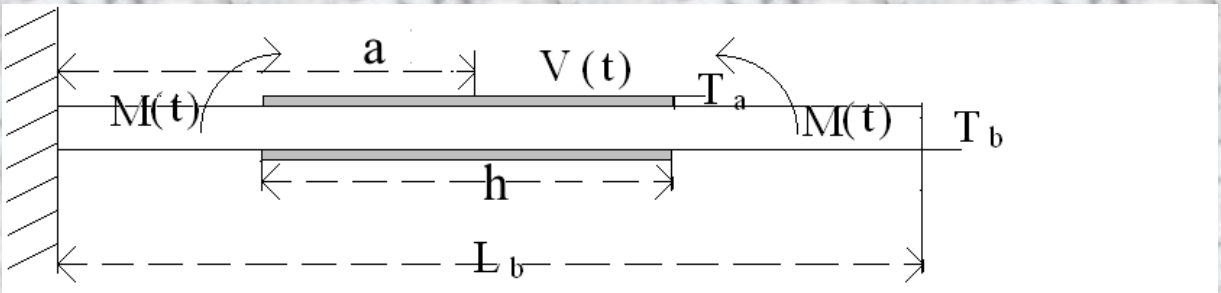
After a transitory time, the amplitude of the vibrations of the free end is:

$$|w(a,h,L)| = \frac{M_a}{a}\left\{\left|\frac{V_i\left[f_i'\left(a+\frac{h}{2}\right)-f_i'\left(a-\frac{h}{2}\right)\right]f_i(L)}{w_i}\right| + \left|\frac{V_j\left[f_j'\left(a+\frac{h}{2}\right)-f_j'\left(a-\frac{h}{2}\right)\right]f_j(L)}{w_j}\right|\right\}$$

If r is the ratio percentage of the j-mo mode:

$$|w(a,h,L)| = \frac{M_a}{a}\left\{\left|\frac{(1-r)\left[f_i'\left(a+\frac{h}{2}\right)-f_i'\left(a-\frac{h}{2}\right)\right]f_i(L)}{w_i}\right| + \left|\frac{r\left[f_j'\left(a+\frac{h}{2}\right)-f_j'\left(a-\frac{h}{2}\right)\right]f_j(L)}{w_j}\right|\right\}$$

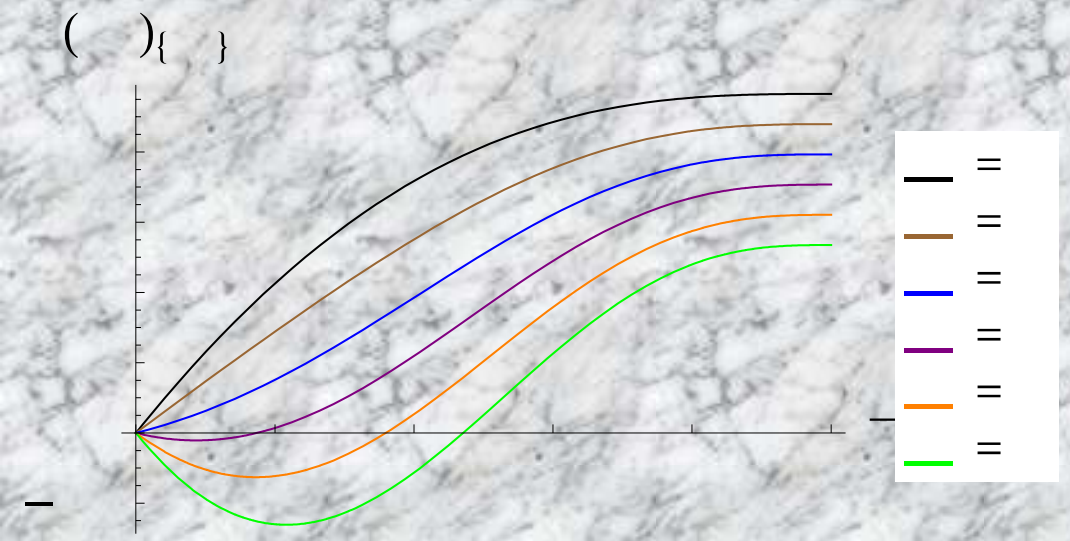
Optimal placement of piezoelectric plates: multimode control



$$a + \frac{h}{2} = L$$

$$\left| w(a,h,L) \right| = w\left(a - \frac{h}{2}, L\right) = w(x,L)$$

$$w(x,L) = \frac{M_a}{a}\left[\left| \frac{(1-r)\left[f_i'(L) - f_i'(x) \right] f_i(L)}{w_i} \right| + \left| \frac{r\left[f_j'(L) - f_j'(x) \right] f_j(L)}{w_j} \right| \right]$$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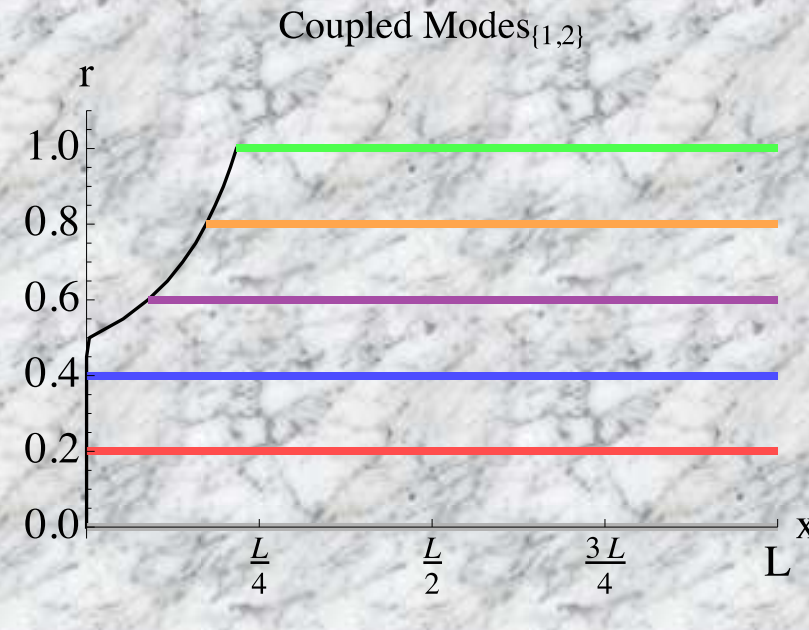
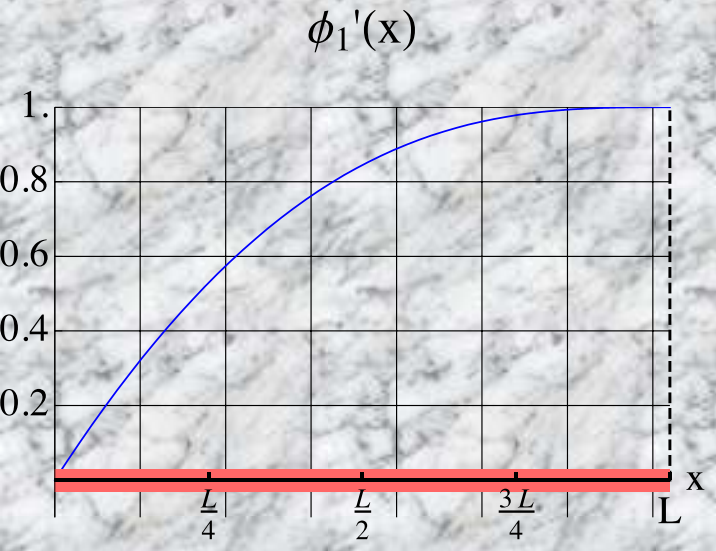
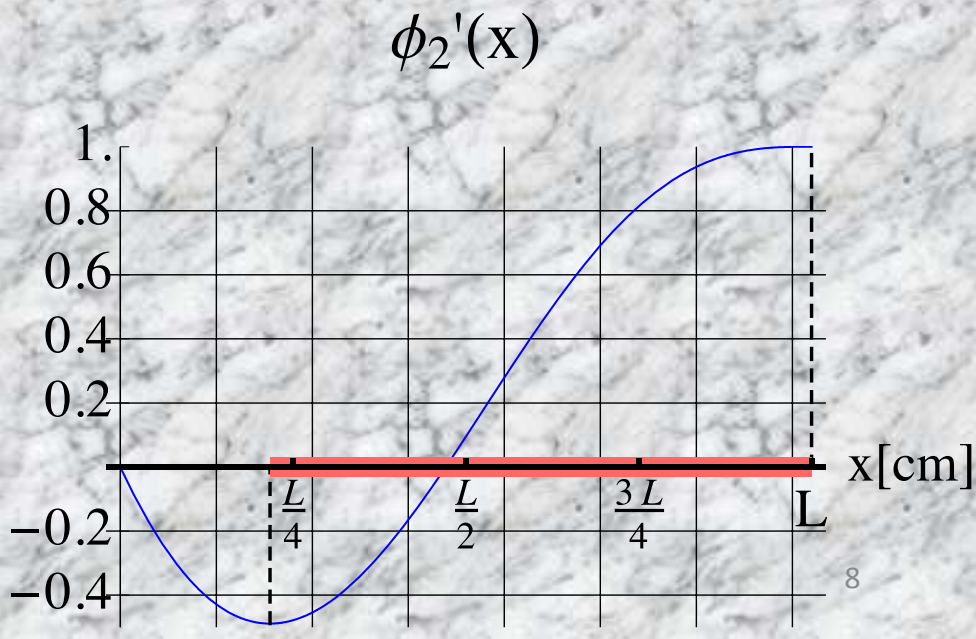
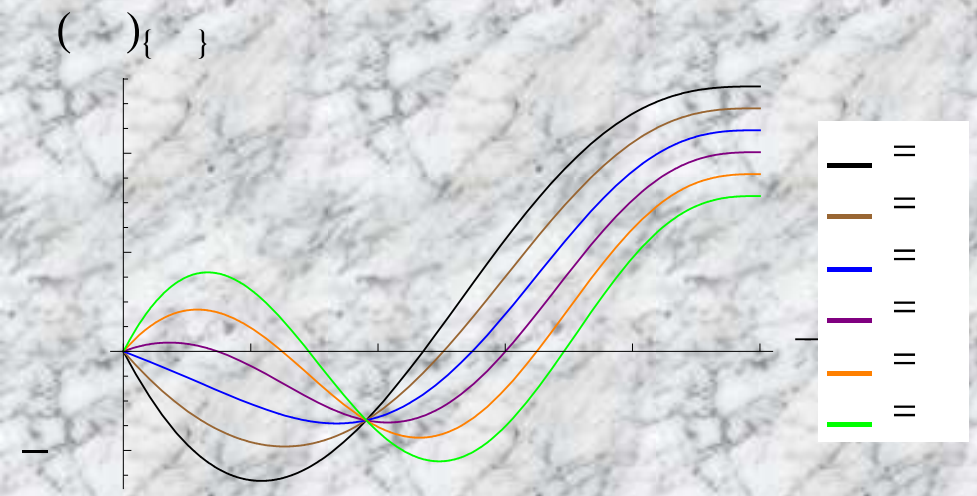
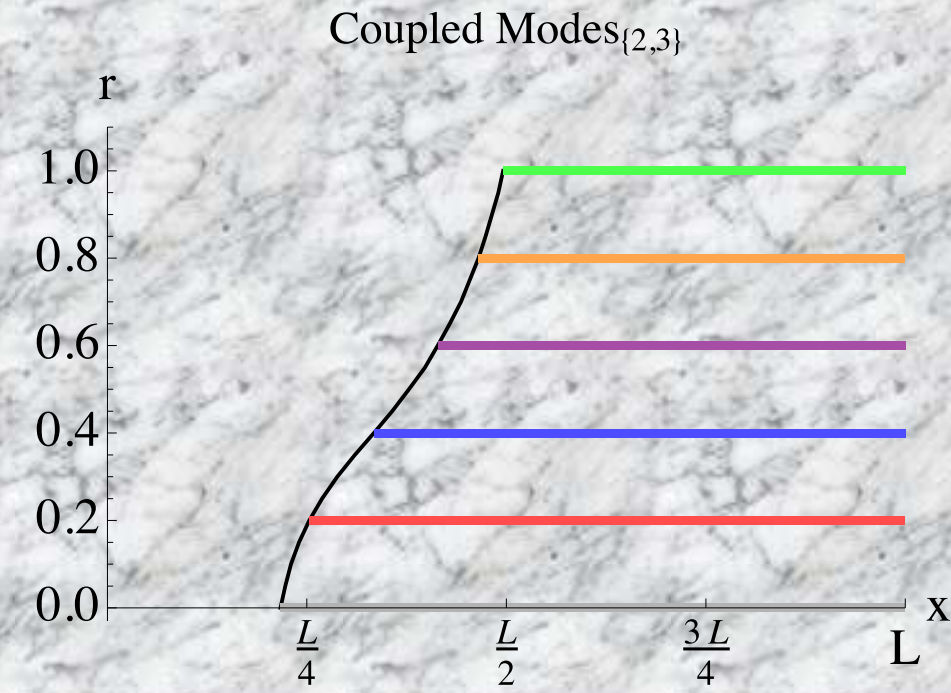

Coupled Modes{1,2}
r
1.0
0.8
0.6
0.4
0.2
0.0
L/4
L/2
3L/4
L
x

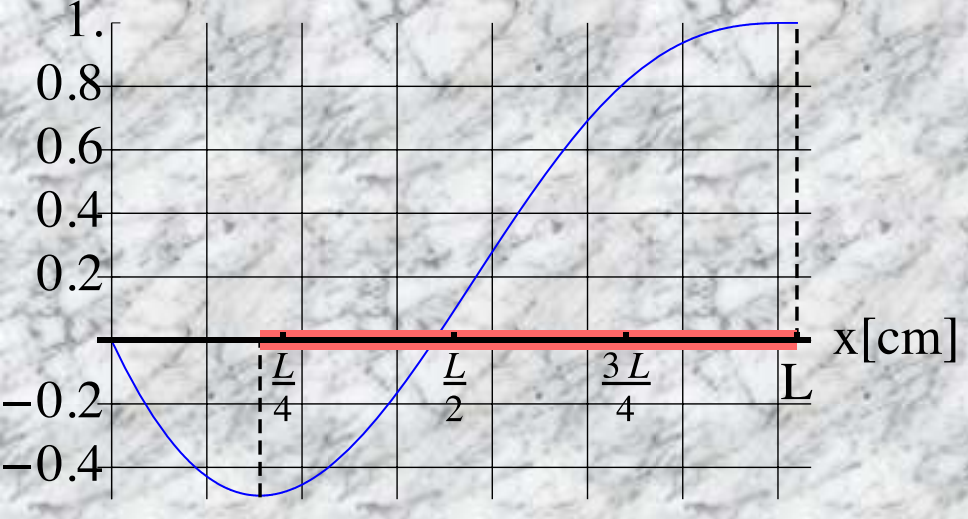

ϕ1'(x)
1.
0.8
0.6
0.4
0.2
L/4
L/2
3L/4
L
x


ϕ2'(x)
1.
0.8
0.6
0.4
0.2
−0.2
−0.4
L/4
L/2
3L/4
L
x[cm]

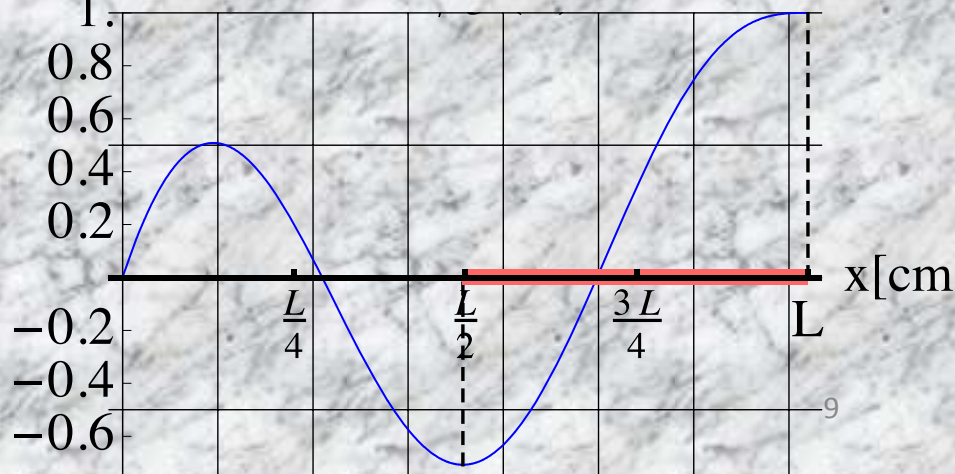



Coupled Modes{2,3}
r
1.0
0.8
0.6
0.4
0.2
0.0
L/4
L/2
3L/4
L
x

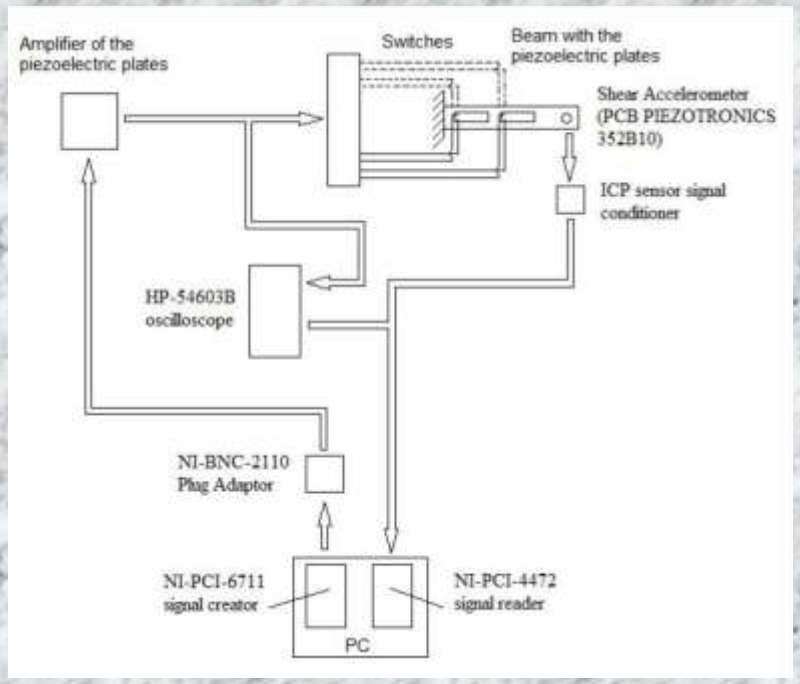
$\phi_2'(x)$

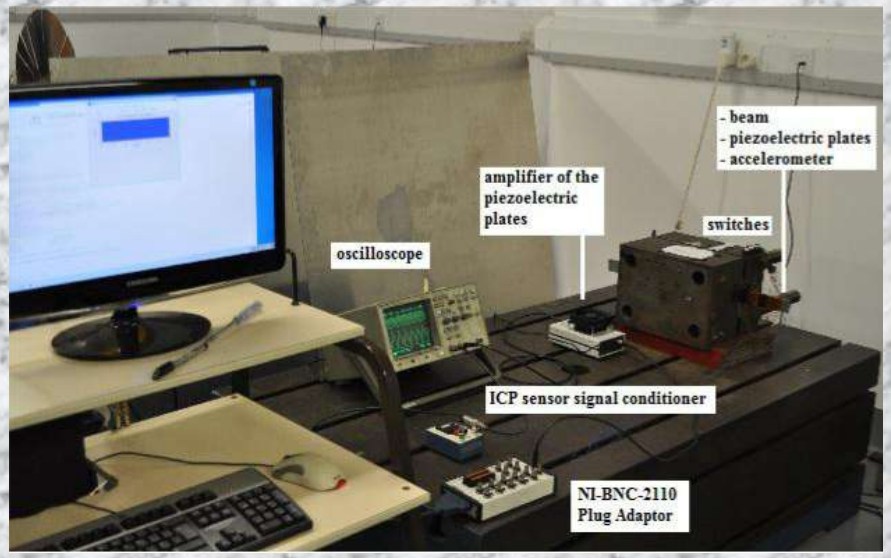

1.
0.8
0.6
0.4
0.2
−0.2
−0.4
L/4
L/2
3L/4
L
x[cm]

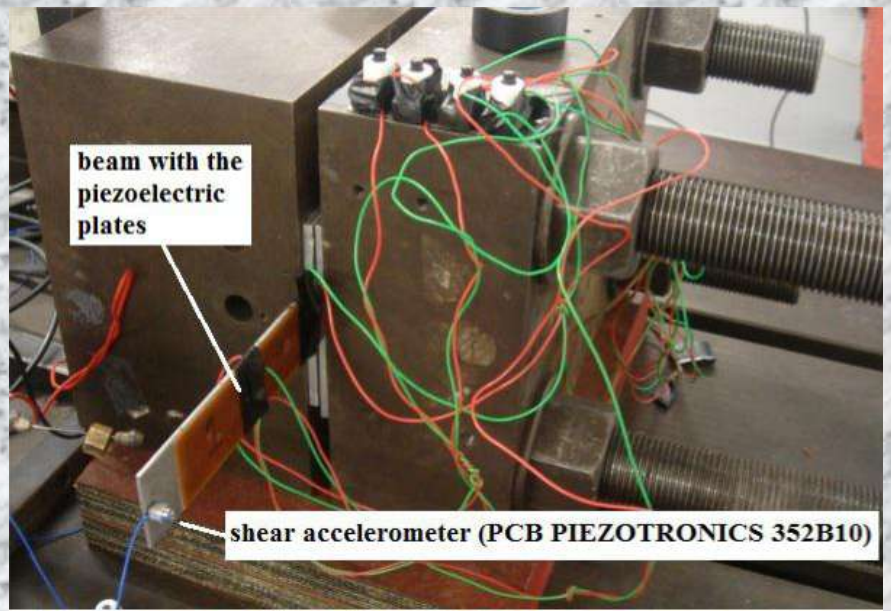
$\phi_3'(x)$


1.
0.8
0.6
0.4
0.2
−0.2
−0.4
−0.6
L/4
L/2
3L/4
L
x[cm

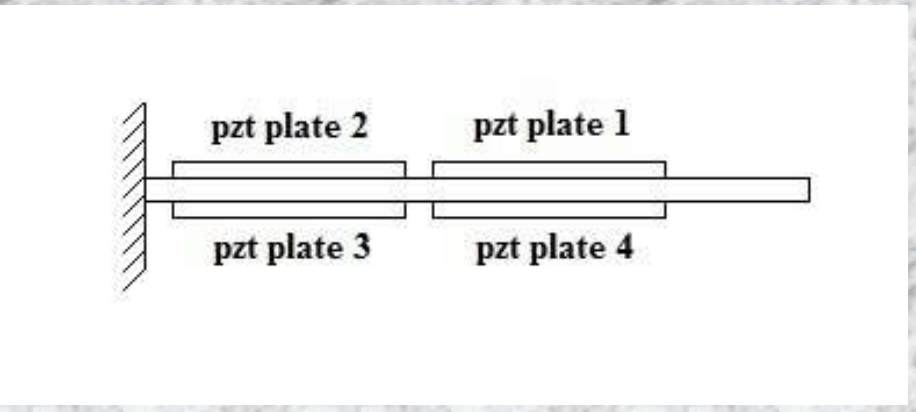
Optimal placement of piezoelectric plates: experimental apparatus for fixing beam







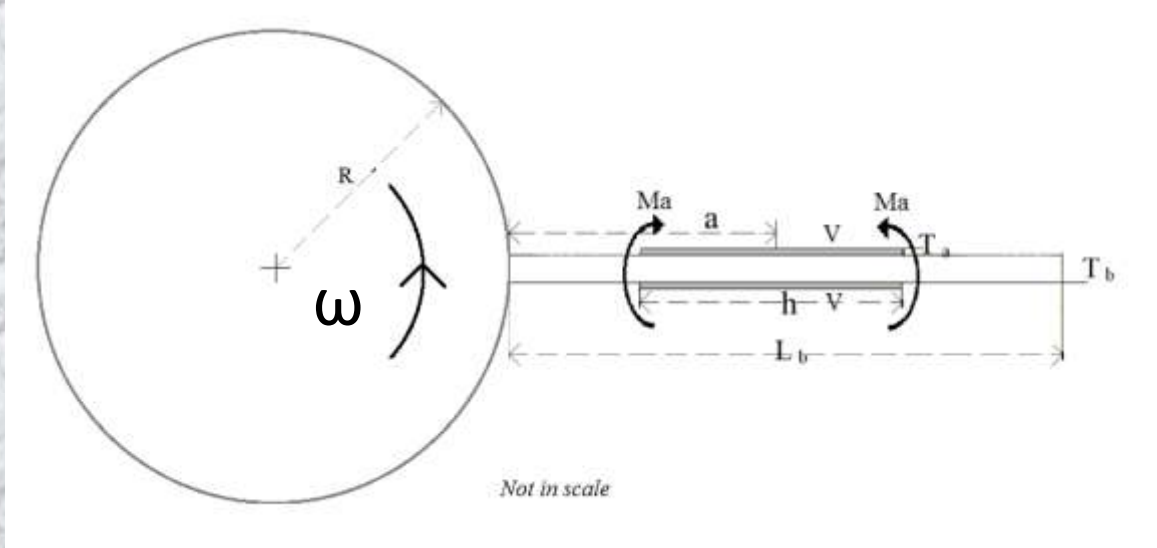
Optimal placement of piezoelectric plates: experimental apparatus for fixing beam

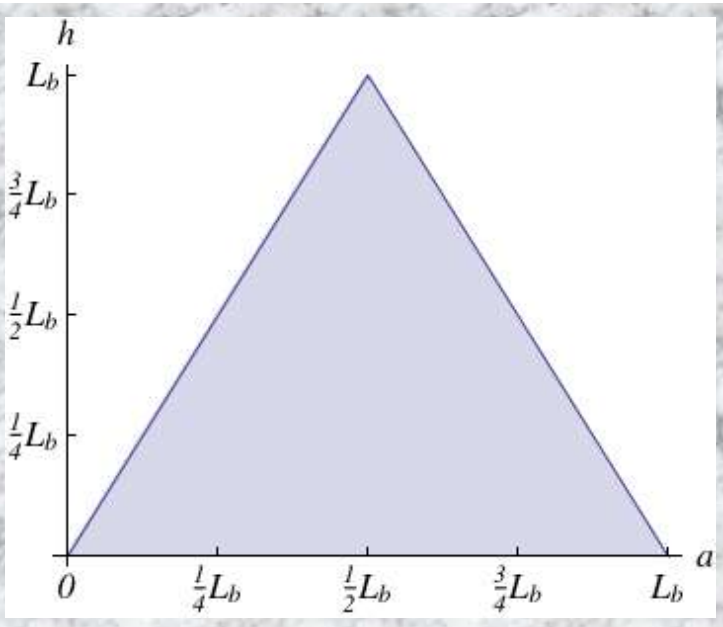


It is possible to find all the experimental results in:

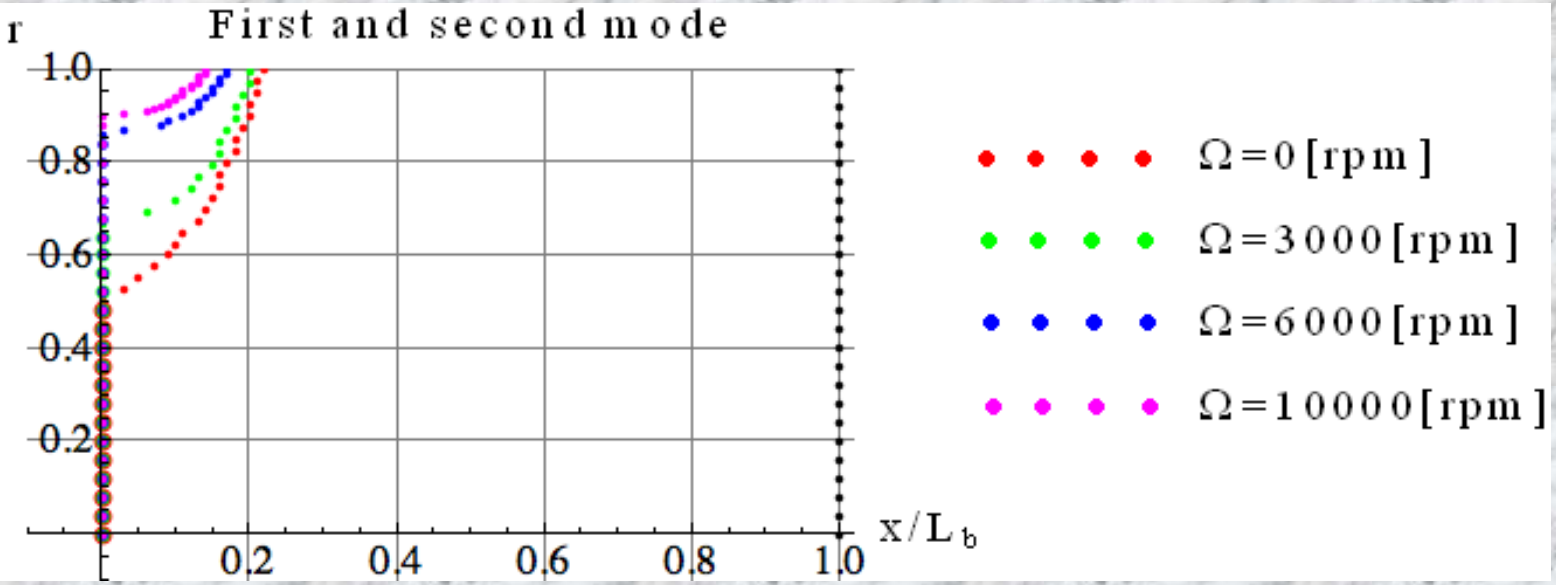
F. Botta, N. Marx, S. Gentili, C. W. Schwingshackl, L. Di Mare, G. Cerri , D. Dini, ***Optimal placement of piezoelectric plates for active vibration control of gas turbine blades: experimental results***, Proc. of SPIE Vol. 8345, 83452H-1- 83452H-11 (San Diego -2012)

Optimal placement of piezoelectric plates: rotating beams

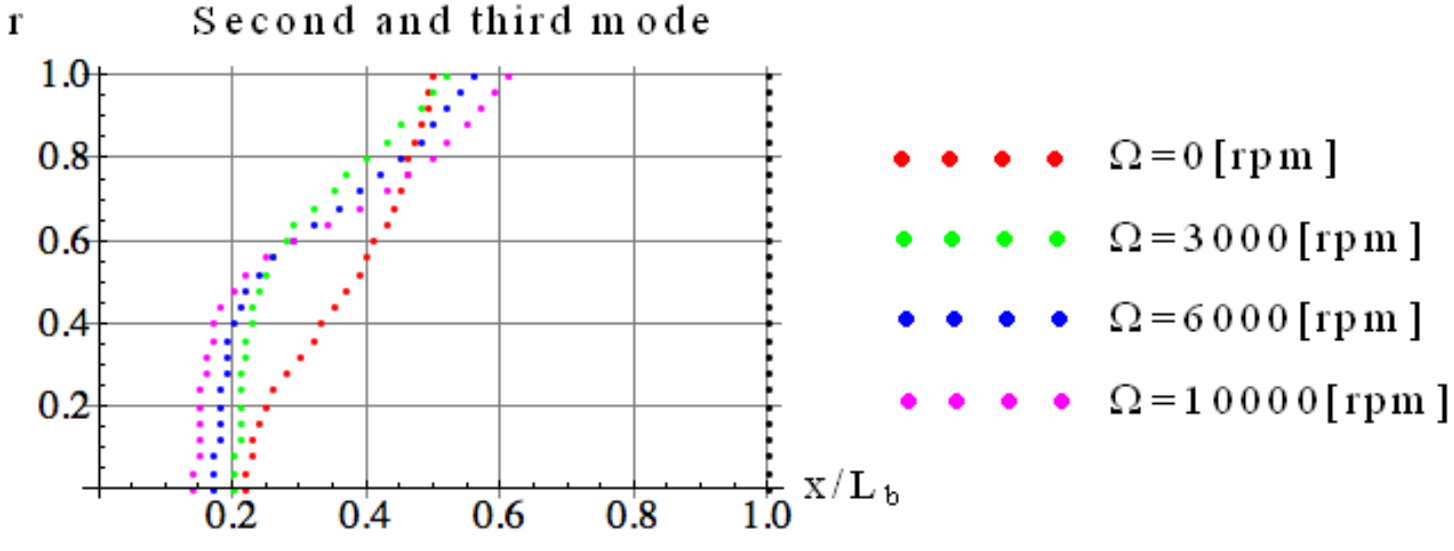




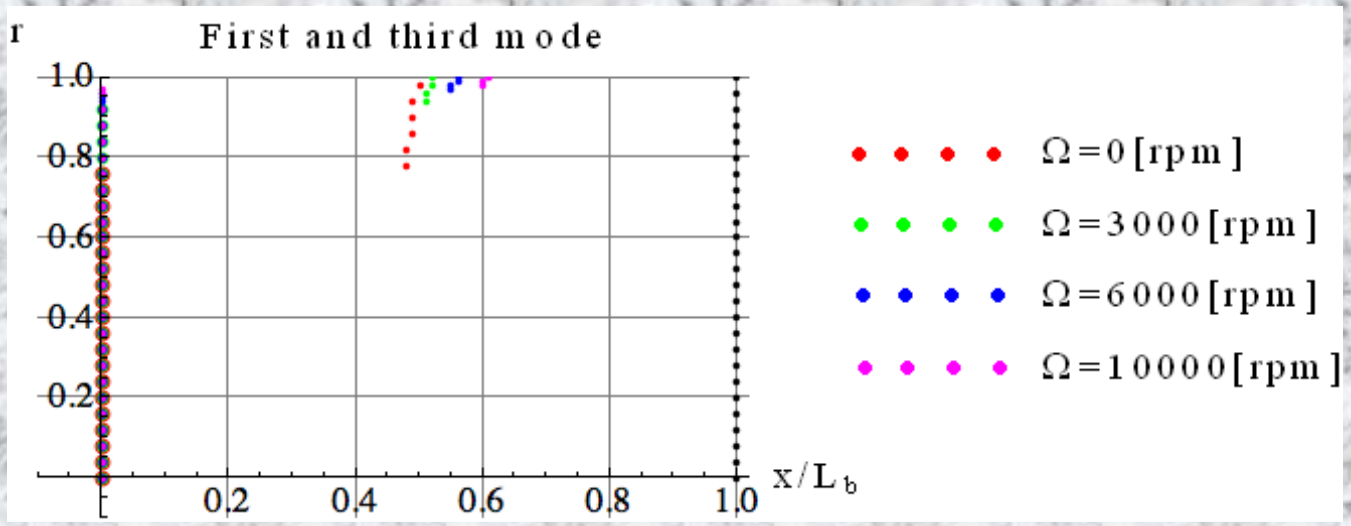
Optimal placement for coupling of the first and second mode



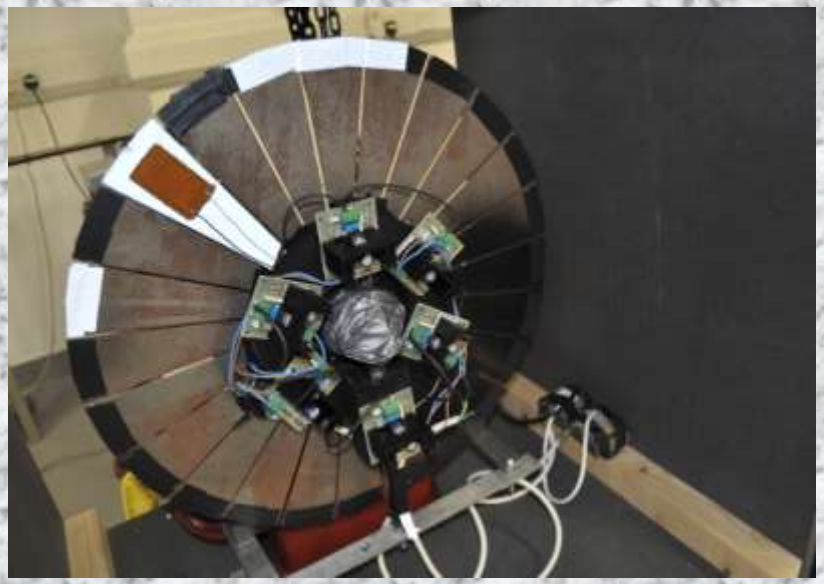
Optimal placement for coupling of the second and third mode



Optimal placement for coupling of the first and third mode



Optimal placement of piezoelectric plates: experimental apparatus for rotating beam



Conclusions and future work

- The optimal placement of piezoelectric plates to control multimode vibrations of rotating beam has been studied.
- Optimal configurations have been reported for different angular velocities and different mode ratio
- Experimental prototype has been built and the experimental tests are going on
- The work must be extended to:
  - more than 2 frequencies
  - torsion effect (numerical studies are going on)
  - real geometry of the blade