

# Design and simulation of piezoelectric energy harvesting system

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# Overview

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- Design of piezoelectric energy harvesting system
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# Introduction

- Conventional batteries used in wireless sensors and portable electronics have limited operating life, contaminate and difficult to replace.
- Energy harvesting is an alternative source of energy for conventional batteries
- Energy harvesting is a process that captures small amounts of energy that would be lost as heat, light, sound, vibration or movement.
- Solar energy is limited in dark conditions.
- Vibrational energy can be harvested through different technologies including piezoelectric ,inductive, electrostatic, thermoelectric devices.
- Piezoelectric materials are used to directly convert mechanical(vibrational) energy to electrical energy.

# Piezoelectric energy harvesting system

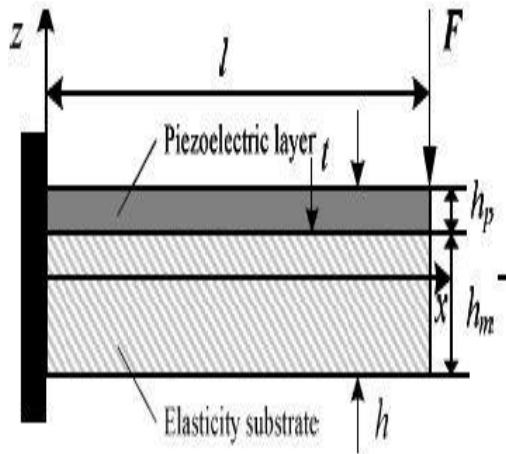
Piezoelectric energy harvesting can be done in different structures which include

- Cantilever type
- Cymbal type
- Stack type
- Shell type

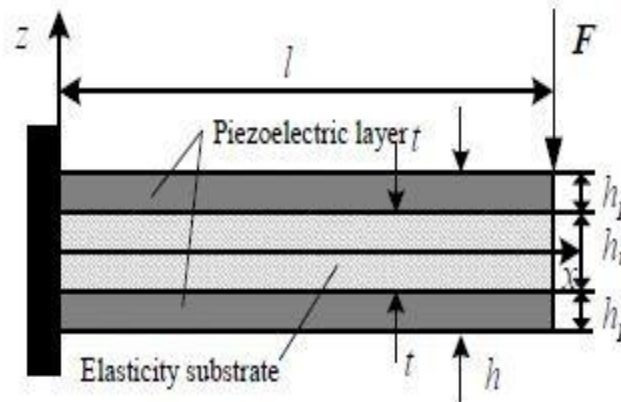
A cantilever type energy harvesting has very simple structure and can produce large deformation under vibration. Thus cantilever type configuration is chosen

There are different types of cantilever configurations that can be used for energy harvesting. These configurations include

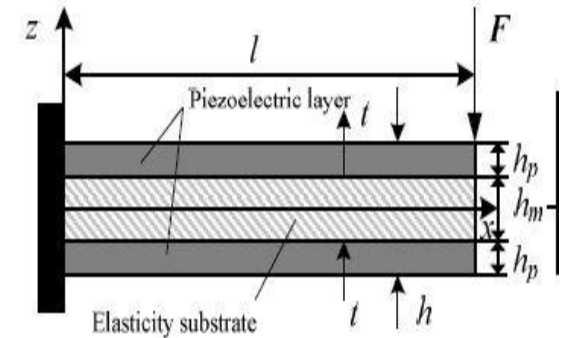
- Unimorph
- Bimorph series
- Bimorph parallel



(a) Unimorph

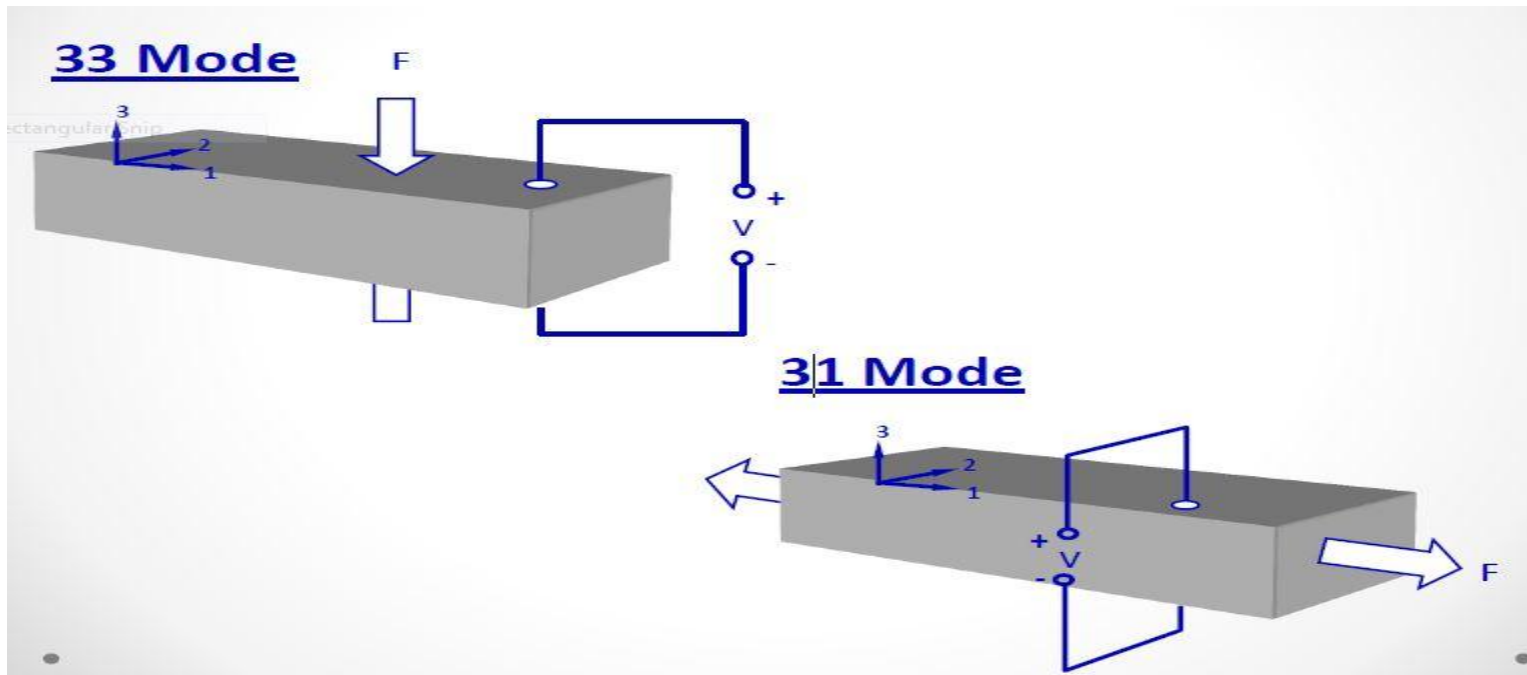


(b) Bimorph series



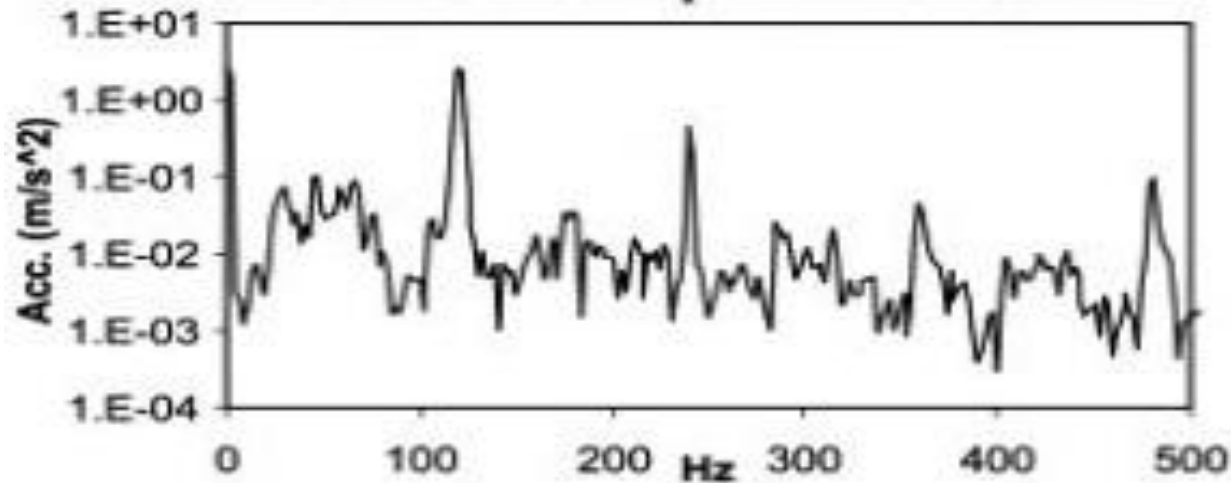
(c) Bimorph parallel

The cantilever model can be used in two different modes, 33 mode and 31 mode.



The most useful mode in harvesting applications is 31 mode, because an immense proof mass is needed for 33 configuration

# Design of piezoelectric energy harvesting system



- The vibration mode above shows that the acceleration decreases for higher modes compared to fundamental mode
- Therefore, the design of cantilever focuses on fundamental mode of frequency

## Unimorph cantilever:

- The power density would be maximum when the vibration frequency matches the resonant frequency of the piezoelectric generator.
- The unimorph cantilever is designed to match the frequency range of environmental vibrations of 60-200 Hz.
- Stainless steel is chosen as substrate and titanium as proof mass
- The dimensions of cantilever are optimized to obtain the frequency of 60 Hz to 200 Hz.



# Theoretical analysis

The frequency of a unimorph cantilever is given by

$$f = \frac{v_n^2}{2\pi} \sqrt{\frac{0.236D_p w}{\left(l - \frac{l_m}{2}\right)^3 (m_s + m_p)}}$$

$$m = \rho_p t_p + \rho_s t_s$$

$$m_s = 0.236mw \left(l - \frac{l_m}{2}\right) + mw \frac{l_m}{2}$$

$$D_p = \frac{(E_p^2 t_p^4 + E_s^2 t_s^4 + 2E_p E_s t_p t_s (2t_p^2 + 2t_s^2 + 3t_p t_s))}{12(E_p t_p + E_s t_s)}$$

The different energy parameters are

$$Q = \frac{-3d_{31}s_s s_p t_s (t_s + t_p) l^2 F}{B} \quad s_s = \frac{1}{E_s}, s_p = \frac{1}{E_p}$$

$$B = s_s^2 t_p^4 + 4s_s s_p t_p^3 t_s + 6s_s s_p t_s^2 t_p^2 + 4s_s s_p t_p t_s^3 + s_p^2 t_s^4 \quad s_h = s_s t_p + s_p t_s$$

$$V = \frac{-3d_{31}s_s s_p t_s t_p (t_s + t_p) l F}{\epsilon_{33}^r w B \left(1 + \left(\frac{3s_p^2 s_s t_p t_s^2 (t_p + t_s)^2}{s_h B} - 1\right) K_{31}^2\right)}$$

$$U = \frac{-9d_{31}^2 s_s s_p^2 t_p t_s^2 (t_s + t_p) l^3 F^2}{\epsilon_{33}^r w B^2 \left(1 + \left(\frac{3s_p^2 s_s t_p t_s^2 (t_p + t_s)^2}{s_h B} - 1\right) K_{31}^2\right)}$$

Where,

$E_p$  = Young's modulus of piezoelectric material,

$E_s$  = Young's modulus of substrate,

$l_m$  = Length of proof mass,

$l = l_b$  = Length of the beam,

$w = w_b = w_m$  = Width of the beam,

$t_p$  = Thickness of piezoelectric material,

$t_s$  = Thickness of substrate,

$m_p$  = Proof mass,

$\rho$  = Density of piezoelectric material,

$\rho_s$  = Density of substrate material.

# Governing equations

- The piezoelectric equations in strain-charge form:

$$S = s^E T + d_t E$$

$$D = d_t T + \epsilon^T E$$

*S* = mechanical strain

*T* = mechanical stress [N/m<sup>2</sup>]

*s*<sup>E</sup> = elastic compliance [Pa<sup>-1</sup>]

*d* = piezoelectric coefficient [C/N]

*D* = electric displacement [C/m<sup>2</sup>]

*E* = electric field [V/m]

$\epsilon^T$  = dielectric permittivity [F/m]

# Design using comsol multiphysics

The different modes of a simply supported cantilever are shown below



Figure III.1. First mode

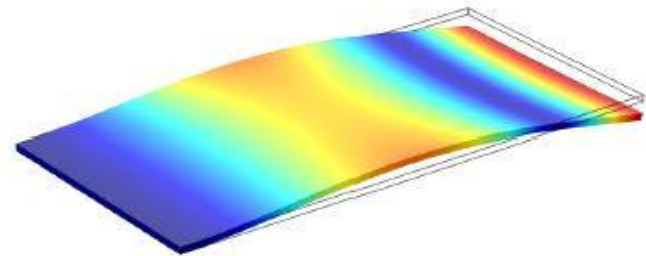


Figure III.2. Second mode

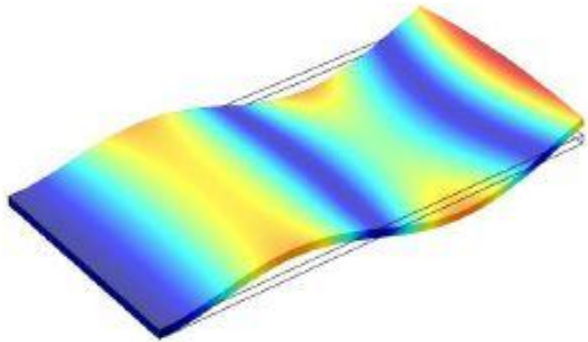


Figure III.3. Third mode

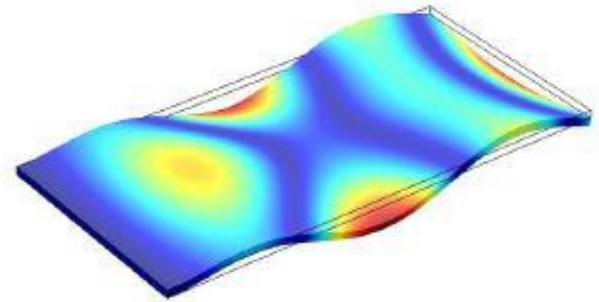


Figure III.4. Fourth mode

The unimorph cantilever is designed in comsol

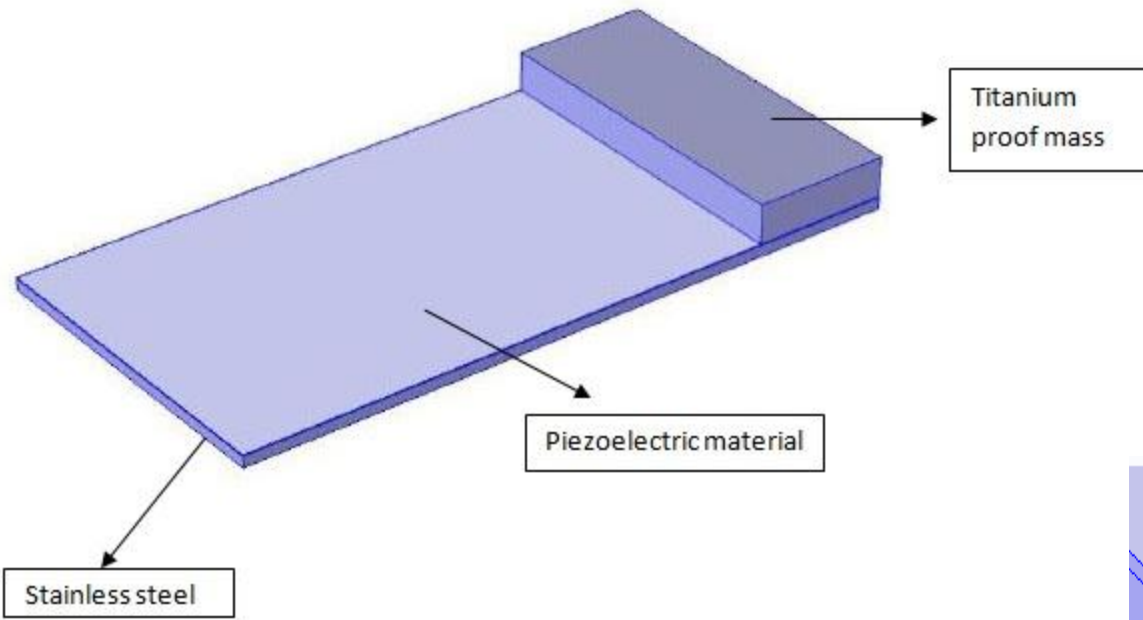
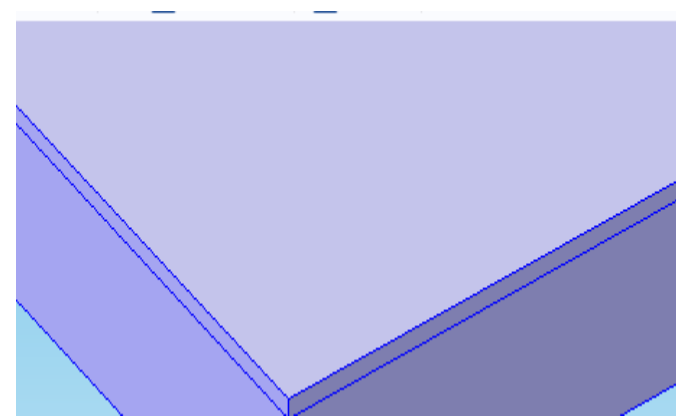


Figure III.5. Designed model in comsol



**PVDF**

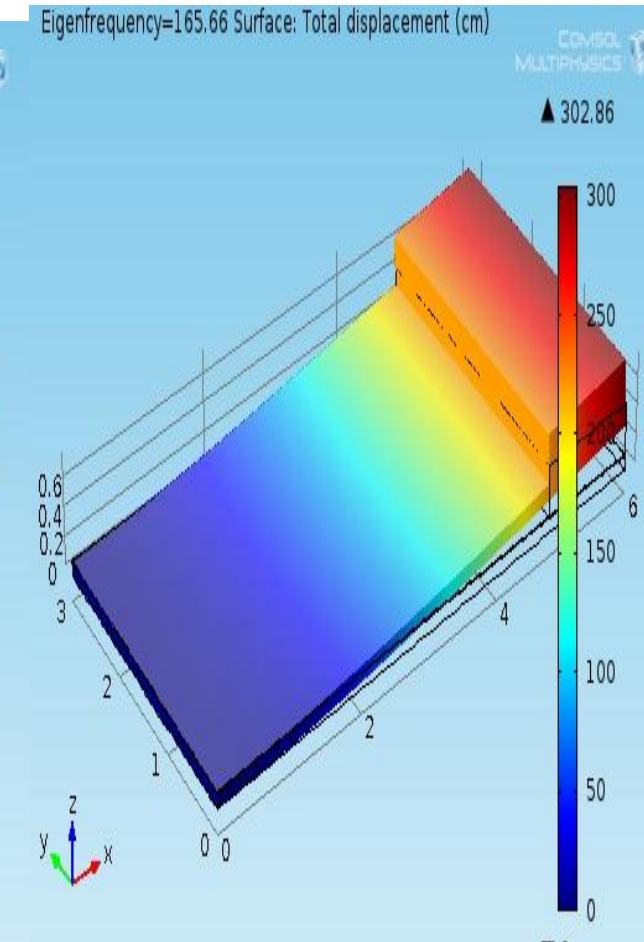
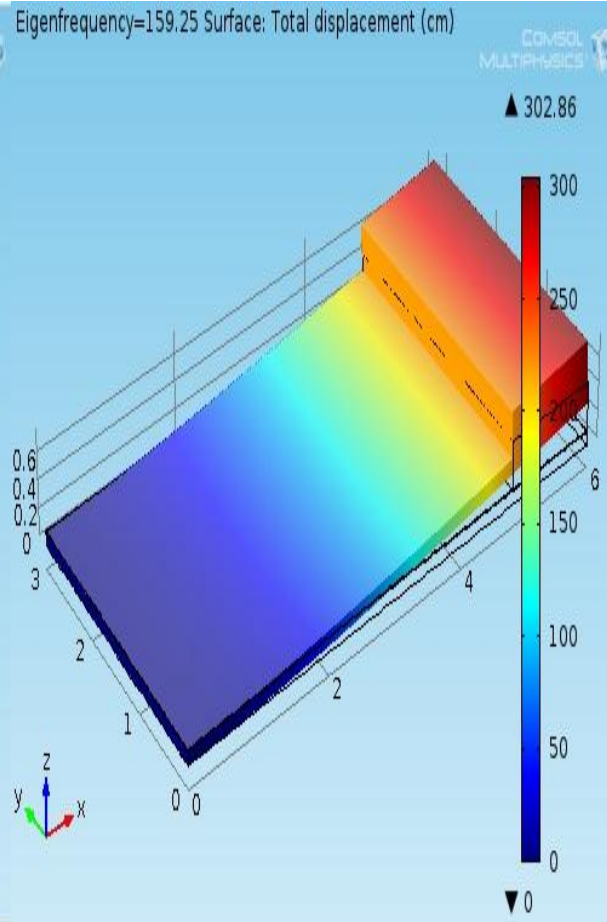
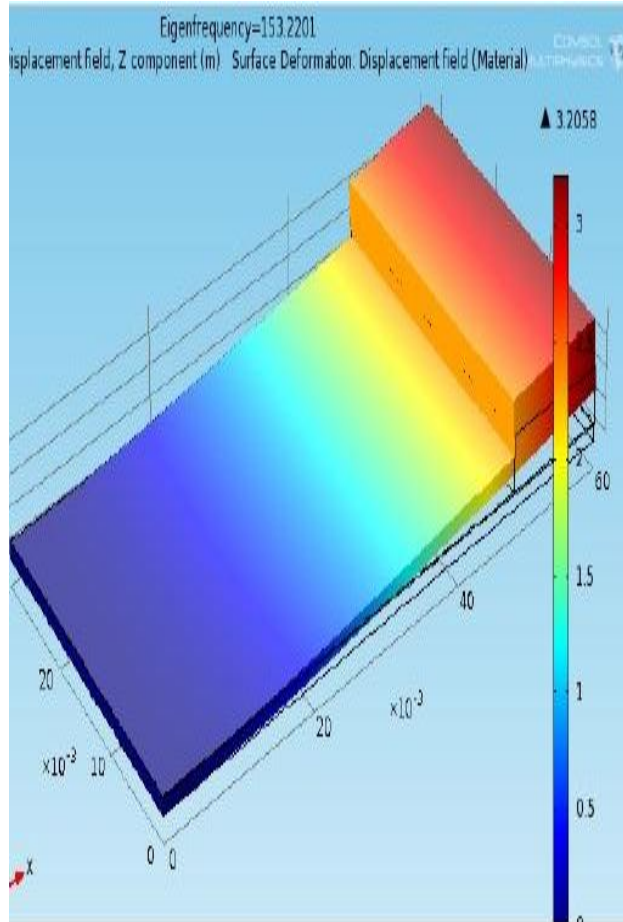
**f=153.22HZ**

**PZT-5H**

**f=159.25HZ**

**PMN-0.33%Pt**

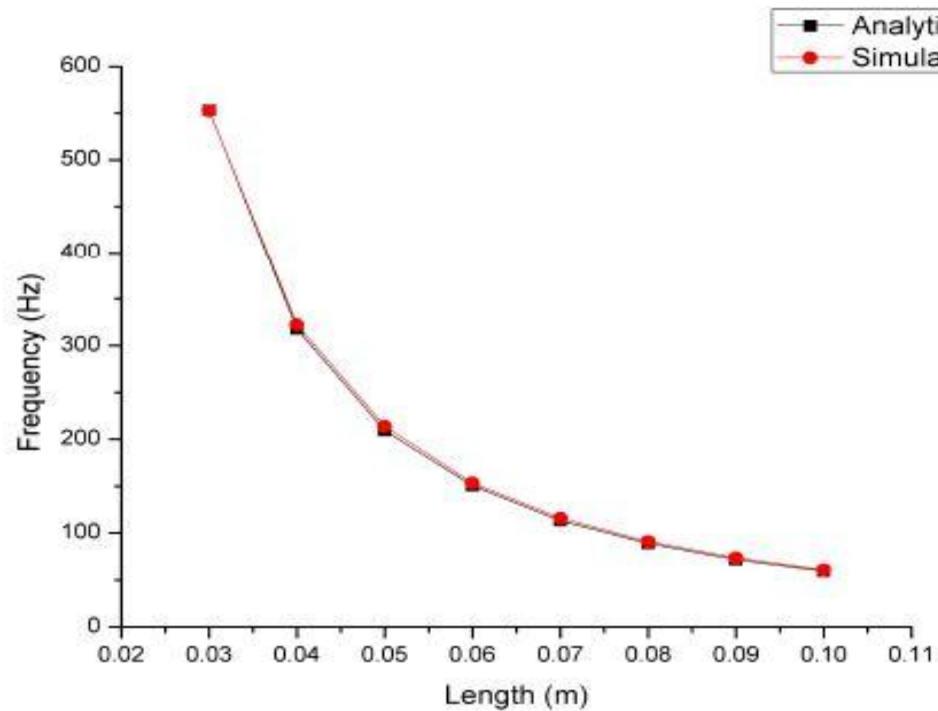
**f=165.66HZ**

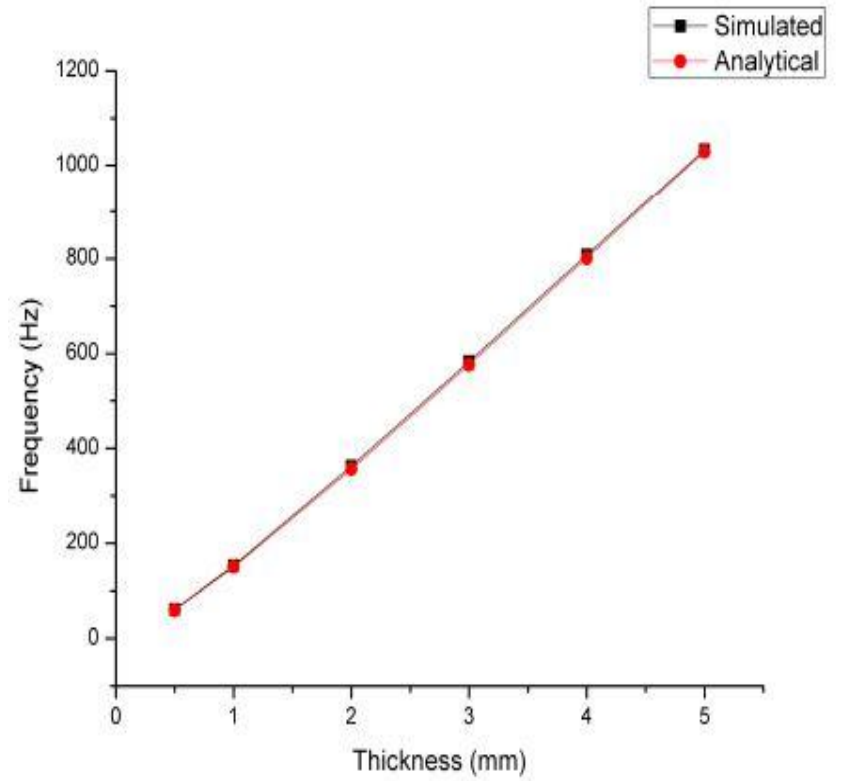
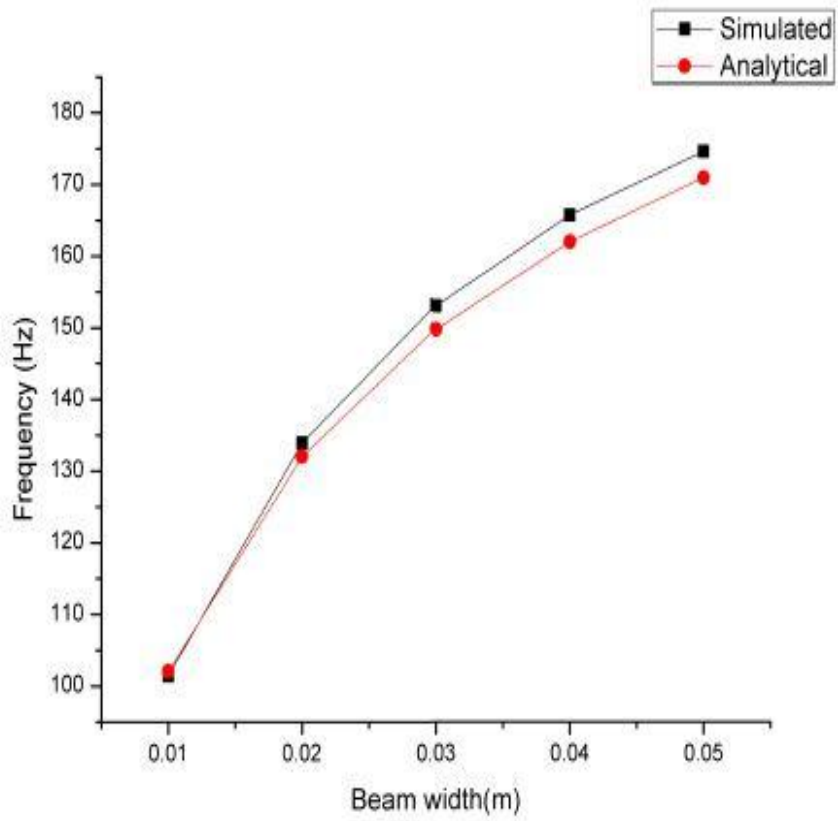




# Results and Discussions

The variation of frequency with design parameters of a unimorph cantilever are shown below



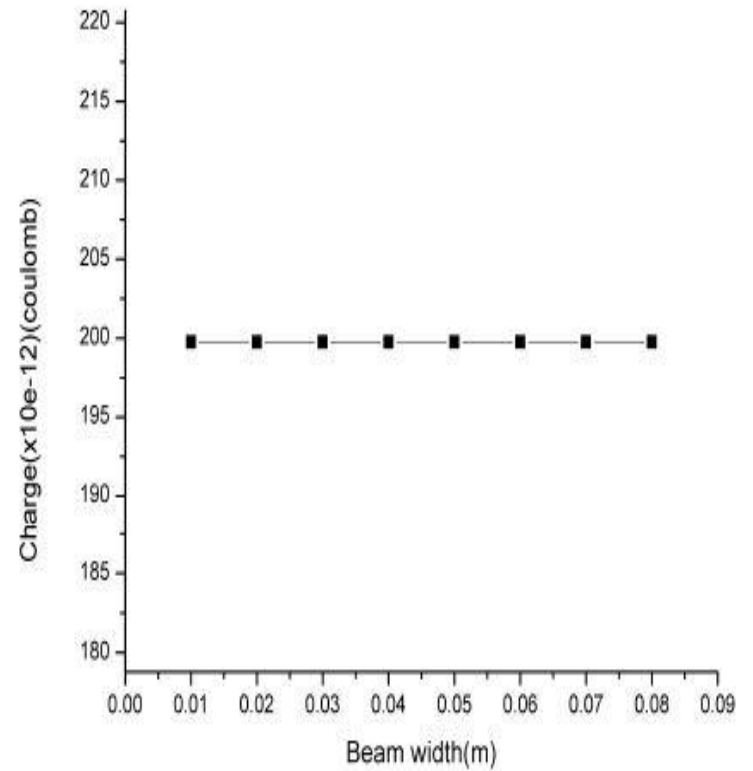
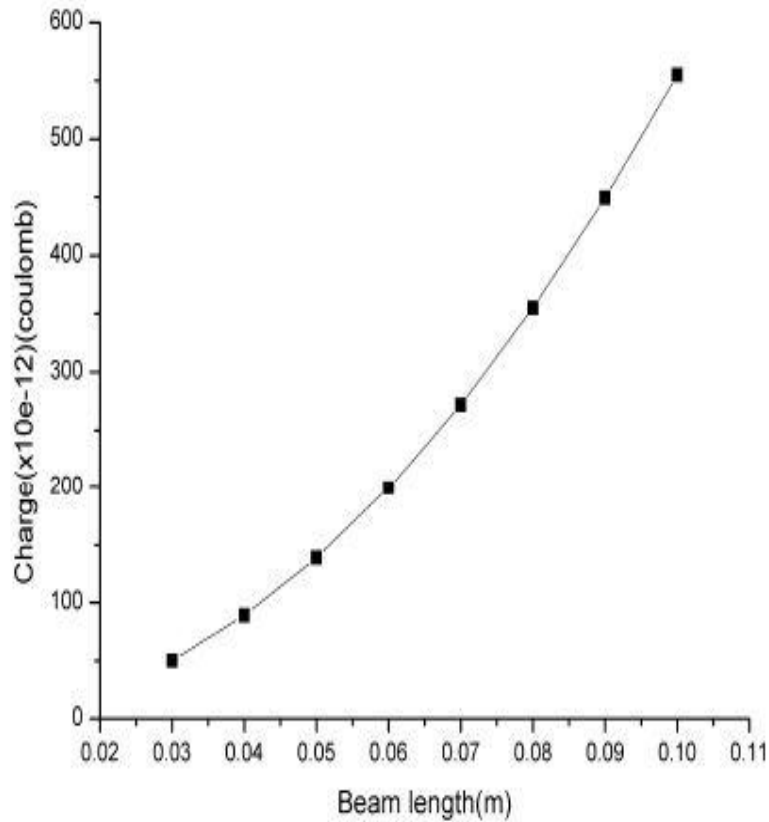


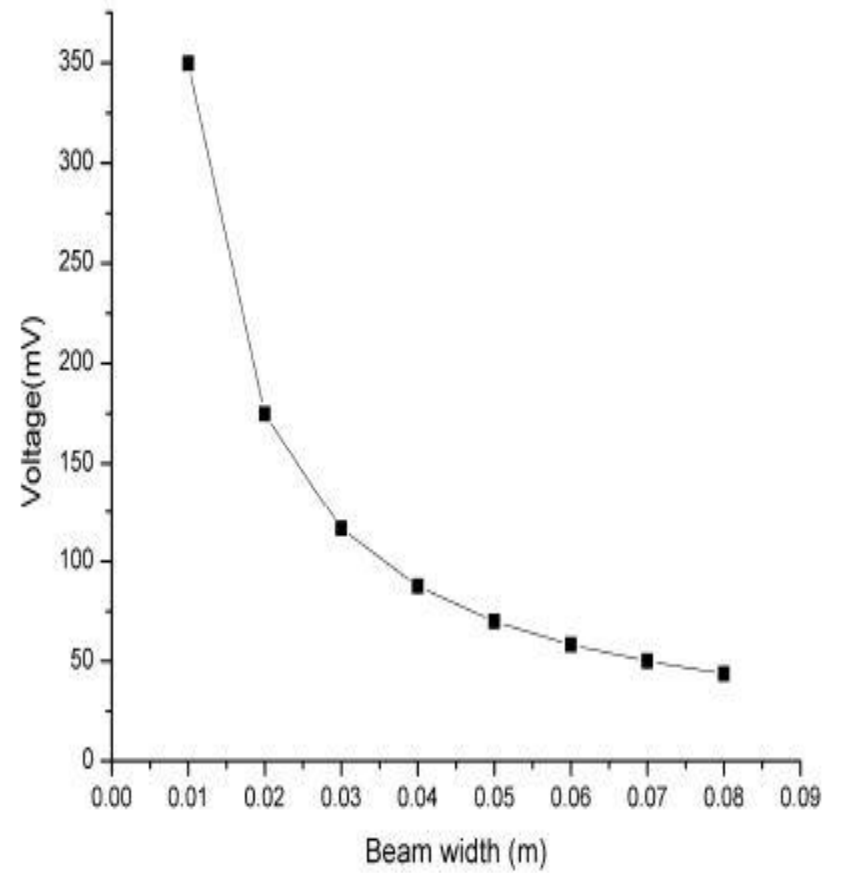
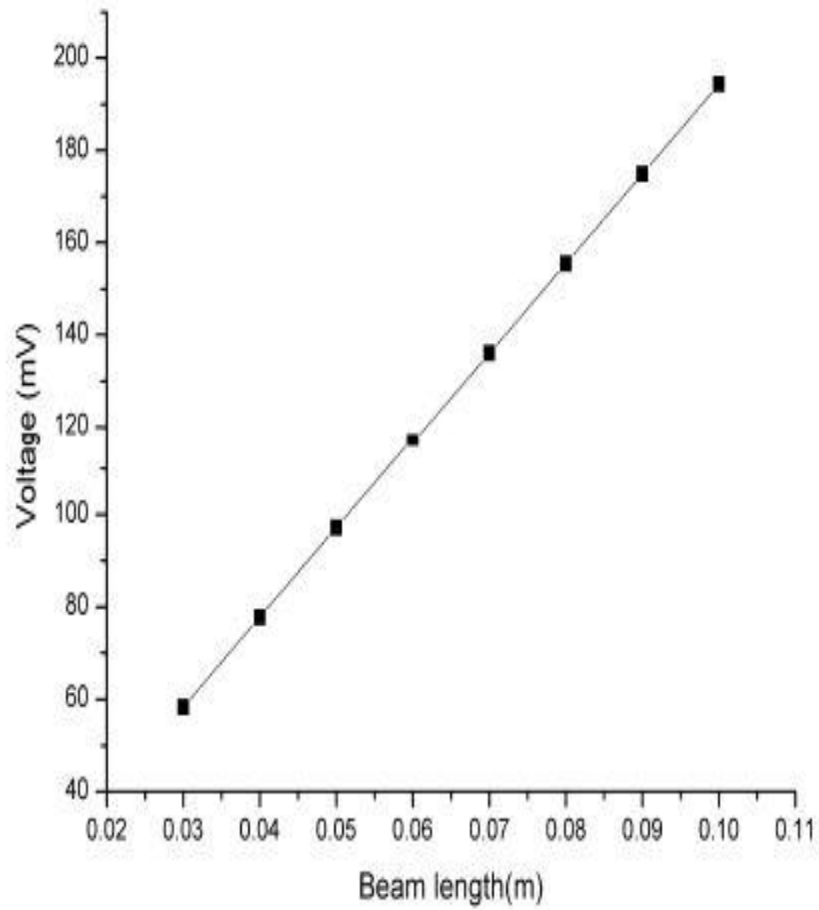
Considering the above parameters, the cantilever with optimized design parameters are shown below

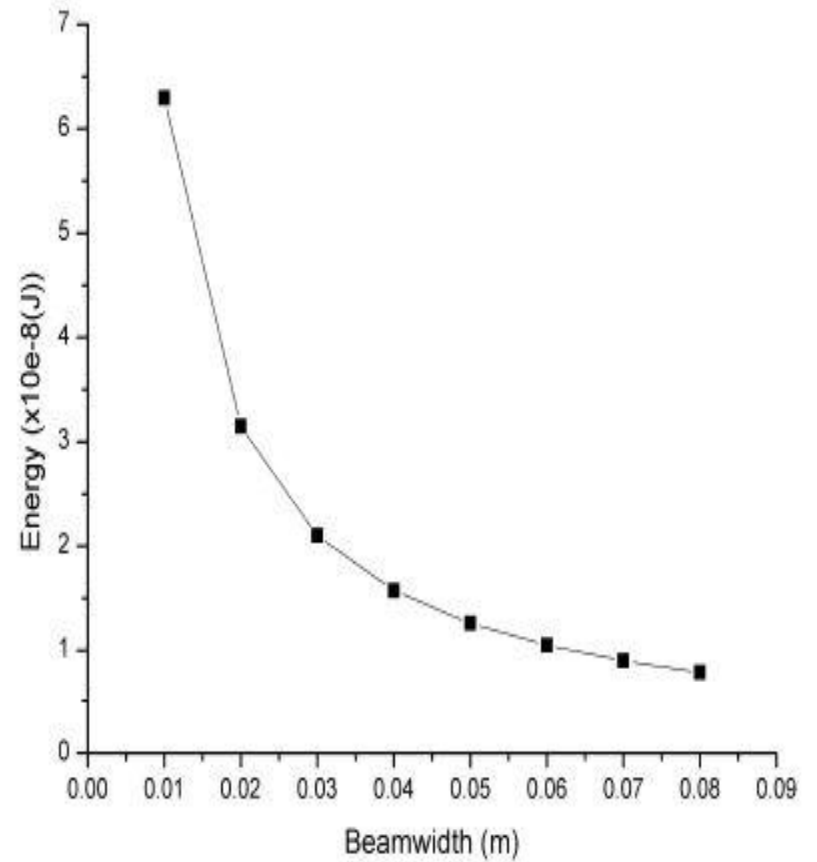
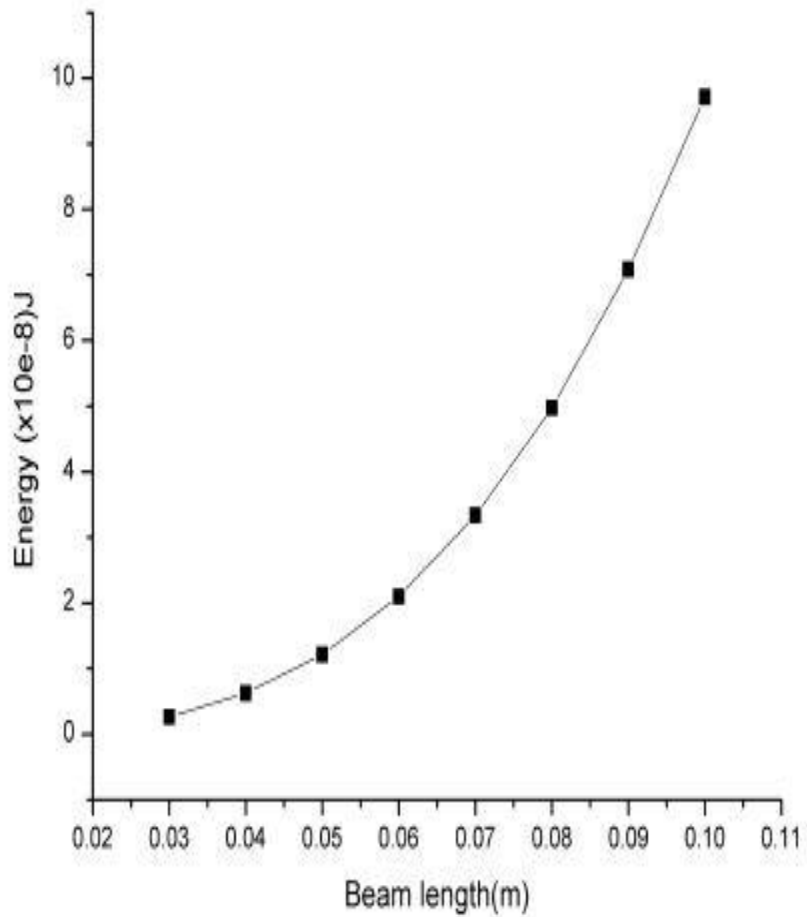
$l_b(cm)$	$w_b(cm)$	$t_b(cm)$	$l_m(mm)$	$w_m(cm)$	$t_m(mm)$
6	3	0.1	12	3	3.5

$E_p(Mpa)$	$E_s(Gpa)$	$t_p(mm)$	$t_s(mm)$	$\rho_p(Kg/m^3)$	$\rho_s(Kg/m^3)$
2450	205	0.11	1	1770	7850

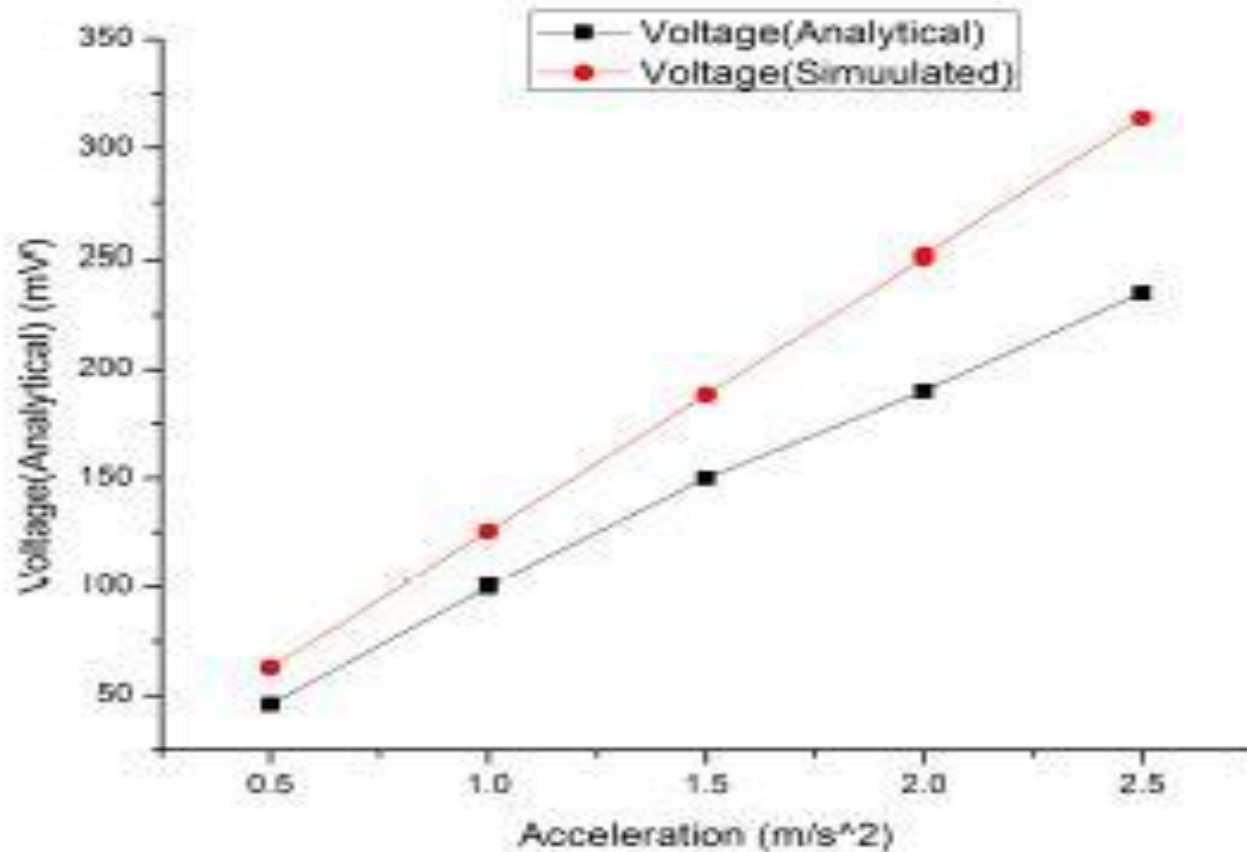
The energy parameters of a unimorph cantilever vary with length, width and height of unimorph cantilever are shown







The following graph shows the comparison between analytical and simulated voltage at different values of acceleration



<b>Material/parameters</b>	<b>PVDF</b>	<b>PZT-5H</b>	<b>PMN-0.33%Pt</b>
Capacitance(Cp)	1.936nF	0.548μF	0.564nF
Charge(Q)	199.12x10e-12 C	52.39x10e-9c	39.17x10e-9c
Voltage(V)	116.6mV	123.2mV	1209.7mV
Energy(U)	2.099x10e-8J	5.815x10e-7J	3.3x10e-5J



# Conclusions

- This work presents the optimization of design parameters of unimorph cantilever for desired frequency
- The variation of charge, voltage and energy with the design parameters of unimorph cantilever is studied
- The comparison is done for different materials and PVDF is found to be an appropriate material for our requirements

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Thank you