Simulation of the Temperature Profile During Welding with COMSOL Multiphysics® Software Using Rosenthal's Approach

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Abstract: A 3D finite element analysis is carried out, using COMSOL software, to reproduce the thermal profile obtained with Rosenthal's equation. The implemented heat transfer equation has been modified as a means to approximate Rosenthal's solution. An analysis of the differences between the simulation and Rosenthal's solution, when the geometry of the domain and the source are changed, has been performed. In addition, the significance of temperature dependent thermal coefficients k and C_p has been studied. A way to model the temperature profile based on Rosenthal's approach is proposed, as well as a strategy to model the thermal profile during tandem arc welding.

Keywords: Rosenthal, welding, quasi-steady state, heat transfer.

1. Introduction

Rosenthal's solution [1] is widely used in welding, especially for the estimation of t_{85}, which is the time needed at a particular position in the base metal to go from 800°C to 500°C. It is an analytical solution, providing a first approximation of the thermal history of the base metal during welding.

The goal of the present work is to explore, using a numerical model, how good of an approximation is the Rosenthal equation. The effect of temperature dependent thermophysical properties and the use of tandem welding are explored.

The approach is to simulate numerically the same problem solved by Rosenthal. The limitations of Rosenthal's model are well known. So by building a more complex model, one can assess the effect of a given parameter on the temperature profile of the weld and the departure from Rosenthal's model. To accomplish this goal, some modifications had to be done to the weak form of the heat transfer equation implemented in COMSOL.

Therefore, after comparing Rosenthal's equation to the simulation and studying the effect on the geometry of the block and the source, the effect of a variation in k and C_p with temperature is studied. Ultimately, a way to study welding with two wires, is proposed.

For the sake of brevity and clarity, the various symbols and their values, where applicable, are defined at the end of the paper in Table A.1.

2. Reference geometry

Within this study a rectangular geometry was used. It is displayed in Figure 1. The source is applied on the upper surface along the welding direction, at 80% of half the distance of the geometry along the welding direction, at A. The origin of the axes is placed at the center of the block, at B.

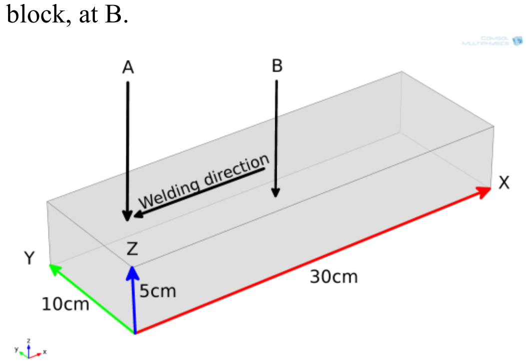

Figure 1. Schematic representing the reference geometry used in this study. The axis and the dimensions of the block are also shown.

3. Equations

3.1 Governing equation

The equation to be solved here is the heat transfer equation:

$$\vec{\nabla}\left(k\vec{\nabla}T\right) + Q = \rho C_p \frac{\partial T}{\partial t} \qquad (1)$$

It is assumed that the domain is isotropic with no volumetric sources and constant thermophysical properties. Equation (1) can then be written as:

$$\nabla^2 T = \frac{\rho C_p}{k}\frac{\partial T}{\partial t} \quad (2)$$

One can define X=x-vt with x being the direction of the weld. In the coordinate system (X,y,z,t)

$$\nabla^2 T = \frac{\rho C_p}{k}\left[\frac{\partial T}{\partial t} - v\frac{\partial T}{\partial X}\right] \quad (3)$$

In this coordinate system steady-state is assumed, so the governing equation is:

$$\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{v\rho C_p}{k}\frac{\partial T}{\partial X} = 0 \quad (4)$$

3.2 Boundary conditions

Rosenthal's model assumes a semi-infinite geometry which means a no flux boundary condition. It is not possible to reach steady-state with numerical software with this boundary condition on all the surfaces and an entering heat flux. Therefore, a convective boundary condition is applied to all the surfaces except the spot corresponding to the source, where an entering flux is considered. The equation used is the following:

$$q_{conv} = h(T - T_\infty) \quad (5)$$

3.3 Initial conditions

The initial temperature (T_0) in the base metal is taken as equal to the ambient temperature (T_∞), i.e., 293.15 K.

4. Rosenthal's equation [1]

Within the scope of this study, the numerical model is compared with Rosenthal's analytical solution. According to Rosenthal, the analytical solution for quasi-steady state in a 3D semi-infinite geometry for a point source is:

$$T = T_0 + \frac{P}{2\pi k}\frac{exp(-\lambda v R)\exp(\lambda v X)}{R} \quad (6)$$

$$\lambda = \frac{\rho C_p}{k} \quad (7)$$

For the case of two electrodes, the numerical solution given by COMSOL is compared with the following superposition of two Rosenthal's solutions:

$$T = T_0 + \frac{1}{2\pi k} \times \left(P_1\frac{exp(-\lambda v R)\exp(\lambda v X)}{R} + \quad P_2\frac{exp(-\lambda v R')\exp(\lambda v X')}{R'}\right) \quad (8)$$

5. Use of COMSOL Multiphysics

5.1 Implementation of the governing equation

To solve the heat transfer equation presented, a 3D steady-state study with the *Heat Transfer in Solids (ht)* module was performed. One problem with this module is that the last term of the equation, due to the moving coordinate system, does not exist. Therefore, a term had to be added manually.

To do so the *Equation View* option is selected, which requires the *Heat Transfer in Solids* module. The second line of the Weak Expressions, which represents the first order terms, is then changed from:

-ht.rho*ht.Cp*
(ht.ux*Tx+ht.uy*Ty+ht.uz*Tz)*test(T) (9)

to

-ht.rho*ht.Cp*
(ht.ux*Tx+<u>v*Tx</u>+ht.uy*Ty+ht.uz*Tz)* (10)
test(T)

The difference has been underlined.

5.2 Post-processing

COMSOL is also used to compare the numerical results obtained by the finite element method with analytical ones presented previously. The absolute relative error in percent is displayed according to:

$$E = 100 \times \left|\frac{T_{calculated} - T_{Analytical}}{T_{calculated}}\right| \quad (11)$$

To make the results more legible the color range for temperature was shortened to [293.15 K-1800 K]. The temperature should not go below the initial temperature and 1800 K is near the fusion point of the studied mild steel. No phase transformations were accounted for, so this part while far from reality is consistent with Rosenthal's assumptions.

5.4 Data

The property data needed to conduct the simulations are mainly taken from Nart and Celik [2]. All the symbols and their values, where applicable, are presented in Table A.1 in the Appendix.

The data were imported from a file into COMSOL. COMSOL is also used to interpolate the data when the values of k and C_p are varied. Only part of this data has been considered. Values near and above the fusion temperature and solid phase transformations have been neglected, because phase transformations are not accounted for. The data are presented in Table 1. Linear interpolation was utilized. Outside of the domain of temperatures covered in Table 1, the values for k and C_p are assumed to be constant.

Table 1: Variation of k and C_p with temperature [2]

Temperature (K)	Thermal conductivity (W/(m K))	Specific heat C_p (J/(kg K))
273	51.9	450
373	51.1	499.2
573	46.1	565.5
623	41.05	630.5
823	34.5	705.5
873	35.6	773.3
993	30.64	1080.4
1023	26	931

6. Results

6.1 One source

First the source was considered as a flux entering the domain via a point source. The problem with this approach is that the density of nodes of the automatically generated mesh is uniform. Therefore, to get rid of some unrealistic results, such as a temperature below the initial one near the source, high node densities had to be used there. The domain was separated into two domains to improve the meshing. The domain was then composed of a rectangular block and a plain hemisphere. However, close to the source and according to Rosenthal's solution, an infinite temperature was encountered. A numerical solution cannot deal with this behavior. Also the post-processing was difficult to justify, as very high and unrealistic temperatures were reached. This made it difficult to display the results correctly.

The plain hemisphere was removed from the slab and the total flux was applied on the free surface. A radius must be selected for the hemisphere. Table 2 compares the different maximum absolute relative errors observed for different hemisphere radii. Rosenthal's approach is widely used to obtain the $t_{8/5}$, which is the time to go from 800°C (1093.15K) to 500°C (793.15K). Therefore, to complement the thermal profile, a surface plot of temperatures ranging from 800°C to 500°C is displayed in Figure 2 along with the absolute error profile. For a radius of 1.10^{-3} m and 1.10^{-4} m most of the domain displays an absolute relative error below 1%.

Table 2: Maximum absolute relative error observed for the reference geometry, an extremely fine mesh, and several radii of the hemisphere

Hemisphere radius (m)	Maximum observed absolute relative error
1×10^{-2}	78.86%
1×10^{-3}	14.5%
1×10^{-4}	11.17% (Figure 2)
1×10^{-5}	25.86%

The effect of size of the domain was first studied. To do so, the length of the domain in each direction (X,y,z) was changed separately.

The X direction (Figure 1) was the first to be changed. This corresponds to the welding direction. When the length of the slab is increased from 30 cm to 60 cm, the temperature on the domain boundaries increases. This breaks the infinite domain assumption used by Rosenthal and results in an increase in the maximum absolute error with Rosenthal's analytical solution. The error increases to 16.62%. When the length in the same direction is changed to 15 cm, the maximal absolute relative error witnessed drops to 5.2%.

Then the y direction was studied. When the dimension is increased from 10 cm to 20 cm, the temperature at the boundaries perpendicular to the y direction is the same as the ambient and the initial one. Therefore, it is closer to the infinite domain assumption. The errors at the edges in this direction are smaller, but the maximum absolute error with Rosenthal's analytical solution stays the same. When the length in y direction is decreased from 10 cm to 5 cm, the temperature at the boundaries perpendicular to the y direction increases and the maximal absolute relative error increases to 28.95%.

The last dimension studied was the z one, which is aligned with the wire feed. When this dimension is changed from 5 cm to 10 cm the temperature at the bottom surface is the initial one, which is in compliance with the assumption of infinity. No difference in the maximum absolute relative error is observed as the largest discrepancies are witnessed along the welding direction. When the dimension is changed from 5 cm to 2.5 cm, the temperature at the bottom surface increases. Therefore, the solution moves further away from the infinite assumption, which triggers a surge in the maximum absolute relative error, increasing to 29.14%.

To explain this behavior, one has to remember that the coordinate system used is a moving one. As such, the axis along the welding direction is equivalent to time. The welding direction is the reverse of the X direction. Therefore as time increases, X increases. When the slab is heated by the source then the temperature increases in the y and z direction by diffusion. Diffusion takes time to occur. So the longer the time after the source passed a given point, the further away along the y and z direction will the thermal effect be felt. When diffusion reaches the boundaries of the domain the infinite assumption is broken. This produces larger discrepancies. However, calculated profiles should always be compared with experimental results.

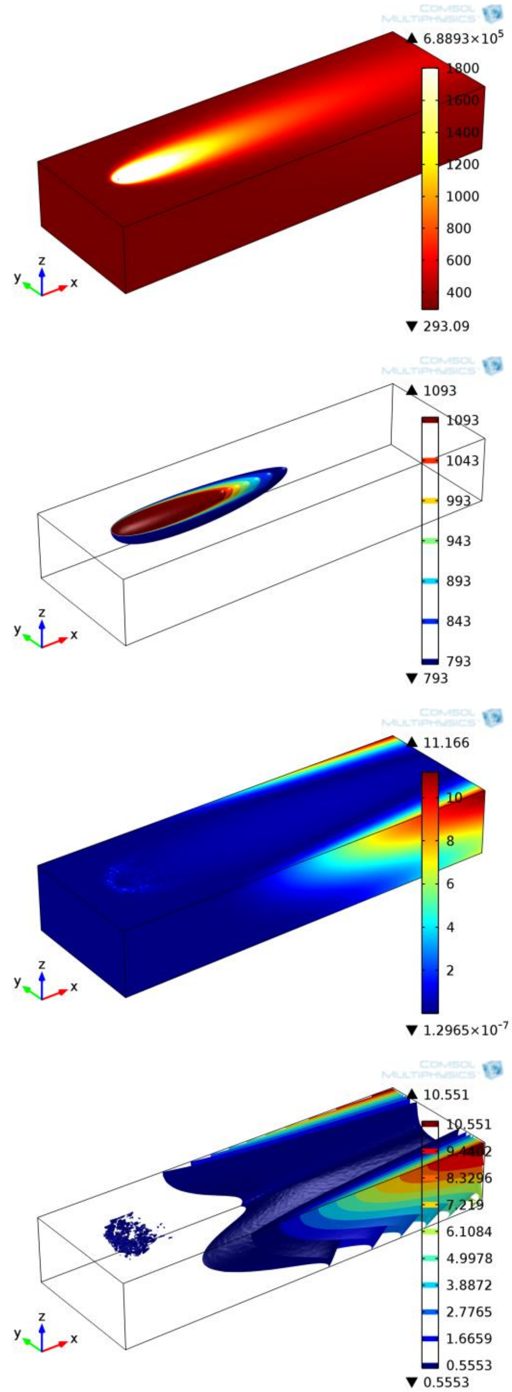

Figure 2. The simulations represent different features for the reference geometry, with the source applied to a 1x10^{-4} m radius hemisphere and a fine mesh. From top to bottom: (a) the temperature profile (K); (b) surface plot of the temperature ranging from 1093.15 K to 793.15 K; (c) the absolute relative error with Rosenthal's model profile; (d) surface plot with the same error.

6.2 One source - k and C_p as functions of T

An example of an application for the COMSOL model is to determine the influence of varying k and C_p on the temperature profile. To do so, some data have been considered and interpolated as explained in Subsection 5.4. The equation solved in this case is the same as Equation (4) except that the value of k changes with temperature. In this case, the reference geometry is used. Figure 3(a) shows the thermal profile obtained by using the same conditions as for Figure 2(a), except that variations in k and C_p have been taken into account. One can notice that the area above 1800 K is narrower and more elongated in the direction of the weld. This may be due to the fact that the C_p increases faster when T increases than k decreases when T increases. Also in Figure 3(b) and 3(c) the absolute relative error with Rosenthal has been represented. One can notice that the maximum for this error is more than 36 time larger than for the case represented in Figure 2. Also, Figure 3(c) shows that the error on most of the domain is larger when the variations in k and C_p are taken into account. Therefore, it seems that variations in k and C_p with temperature have a major effect on the thermal profile in welding.

6.3 Two sources

Good agreement between the numerical solution and the analytical one has been found for one source, with k and C_p assumed constant (subsection 6.1). The approach, with k and C_p constant, was extended to two sources, which are represented by a superposition of two analytical solutions. To determine whether this approach has any validity, a numerical experiment can be performed to provide a first impression on the relevance of this approach.

The temperature profile, as well as the absolute relative difference between the numerical results and the analytical ones, is shown in Figure 4. The difference between the superposition of analytical solutions and the numerical model is bounded by 17.74%. The large differences are close to the boundaries; most of the domain is dark blue in color (Figure 4b), which corresponds to a difference of less than 1%. This approach appears promising and should be compared with experimental results.

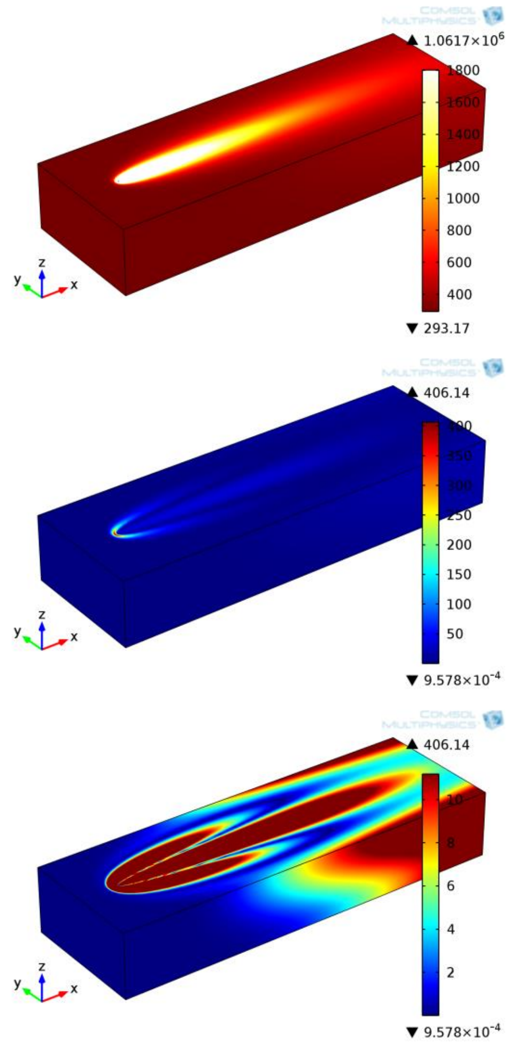

Figure 3. Results obtained for the same conditions as for Figure 2(a), except that variations in k and C_p with the temperature have been taken into account. From top to bottom: (a) the temperature profile; (b) the absolute relative error with Rosenthal's model; (c) the same as (b) but the color range has been shortened to the one of Figure 2(c) to make the comparison easier.

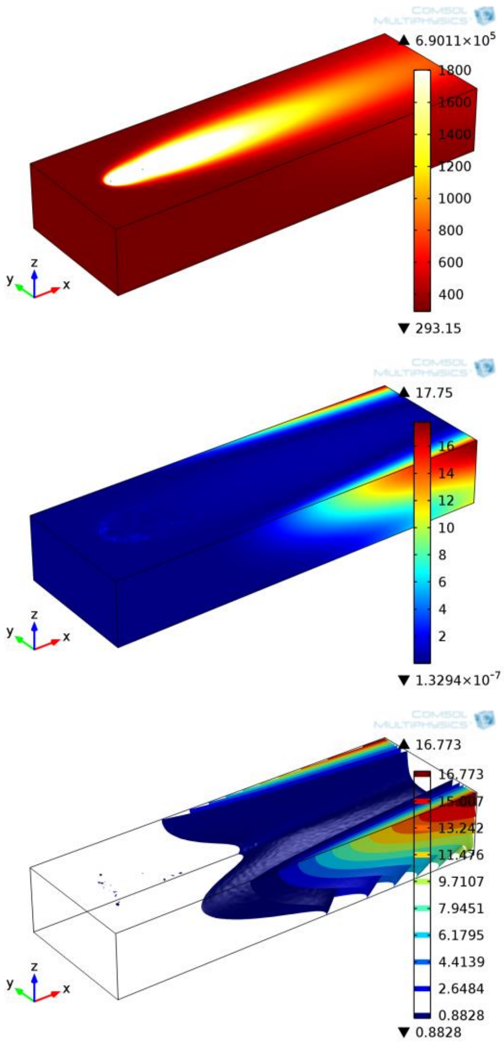

Figure 4. Simulations representing different features for two wire welding, with the reference geometry. Two sources applied as two $1x10^{-4}$ m radius hemispheres separated by 2.5 cm, with the same heat input applied to each source and an extremely fine mesh. k and C_p are assumed to be constant. From top to bottom: (a) the temperature profile in K; (b) the absolute relative difference with the superposition of two Rosenthal analytical solutions; (c) surface plot of the same difference.

7. Conclusions

It has been demonstrated that it is possible to effectively reproduce Rosenthal's solution by using COMSOL. It has also been shown that the coordinate in the welding direction is equivalent to time and that the discrepancy between the two solutions increases as time increases, so that a larger geometry must be considered to stay within the semi-infinite domain assumption.

The influence of variations in k and C_p has been considered using COMSOL and there is a tendency for diffusion to be more directional along the welding direction. Also, these variations seem to have a major effect on the temperature profile during welding.

A method to model the thermal profile for tandem welding, both numerically and analytically, has been proposed.

8. References

1. Rosenthal D., The theory of moving sources of heat and its application to metal treatments, *Trans A.S.M.E.*, **68**, 849-866 (1946)
2. Nart, E. and Celik, Y., A practical approach for simulating submerged arc welding process using FE method, *Journal Of Constructional Steel Research*, **84**, 62-71(2013)

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10. Appendix

Table A.1: List of symbols used

Symbol	Description	value	Units
C_p	Specific heat	600 if constant	J/(kg.K)
h	Convective coefficient	15	$W/(m^2.K)$
k	Thermal conductivity	40 if constant	W/(m.K)
P	Source power	17290	W
P_1	Power from first source	17290	W
P_2	Power from second source	17290	W
Q	Volumetric source	0	W/m^3
q_{conv}	Thermal convective flux	-	W/m^2
R	Distance from first source	-	m
R'	Distance from first source	-	m
t	Time	-	s
T	Temperature	-	K
$T_{calculated}$	Temperature from simulation	-	K
$T_{analytical}$	Temperature from analytical formula	-	K
T_0	Initial temperature	293.15	K
T_∞	Temperature of surrounding atmosphere	293.15	K
v	Travel speed	$6.5.10^{-3}$	m/s
x	Dimension along welding direction in the fixed coordinate system	-	m
X	Same as x, but in moving coordinate system	-	m
X'	Distance between the second source and a point along the X direction	-	m
y	Dimension perpendicular to the other two	-	m
z	Dimension aligned with the wire feed	-	m
λ	Inverse of the thermal diffusivity	118050 if constant	s/m^2
ρ	Density	7870	kg/m^3