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Implementation of a Viscoelastic Material Model to Simulate Relaxation in Glass Transition

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Corning Incorporated

Founded:
1851

Headquarters:
Corning, New York

Employees:
~34,000 worldwide

2013 Sales:
~$8.0 billion

Fortune 500 Rank (2014):
343

- Corning is one of the world’s leading innovators in materials science. For more than 160 years, Corning has applied its unparalleled expertise in specialty glass, ceramics, and optical physics to develop products that have created new industries and transformed people’s lives.

- Corning succeeds through sustained investment in R&D, a unique combination of material and process innovation, and close collaboration with customers to solve tough technology challenges.

Outline

- Introduction
- Backgrounds
  - Stress relaxation
  - Structural relaxation
- Numerical example
- COMSOL implementation
- Results
- Conclusion

Introduction

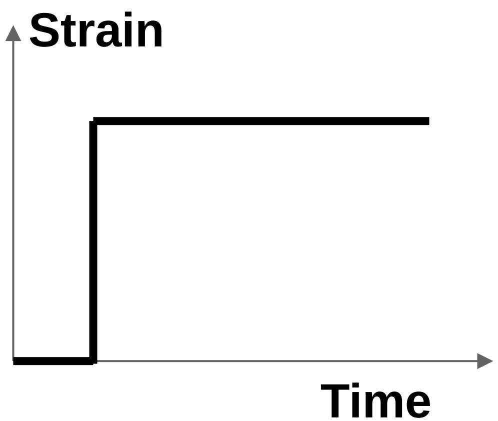
- Glass transition
  - Transform of liquid glass melt to solid-like glass state with cooling
  - Occur around glass transition temperature T_g
  - Dramatic viscosity increasing
  - Material property changes: thermal expansion, heat capacity, …

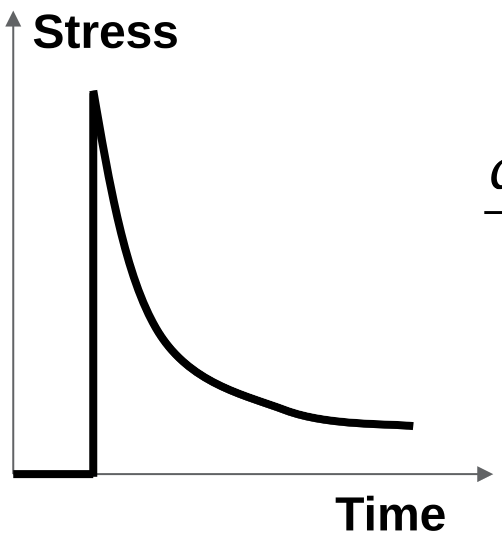
- Characterized by two viscoelastic relaxation phenomena
  - Structural relaxation

    Time-dependent intermolecular rearrangement change (volume, thermal expansion, …) due to temperature change
  - Stress relaxation

    Time-dependent change in dimensions due to applied loadings

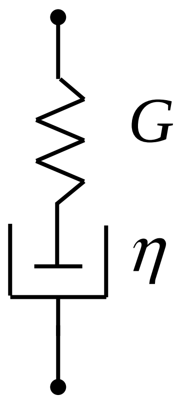
Stress Relaxation





Stress relaxation

Maxwell element

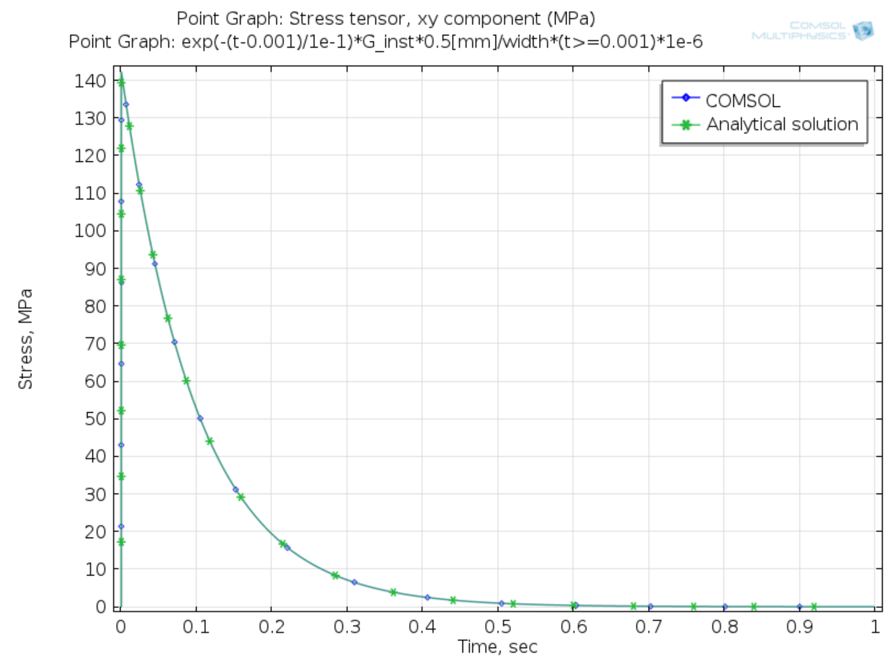


$$\frac{d\sigma}{dt} + \frac{\sigma}{\tau} = 2G\frac{d\varepsilon}{dt}$$

$$\tau = \frac{\eta}{G}$$

$$\tau = \frac{\tau_0}{\Phi}$$

Φ Shift function



$$\sigma(t) = 2G(0)\varepsilon_0 e^{-\frac{t-t_0}{\tau}}$$

Structural Relaxation

- Fictive temperature and Tool's equation (A.Q. Tool)

$$\frac{dT_f}{dt} = \frac{T - T_f}{\lambda} \qquad \lambda = \lambda_0 / \Phi$$

- Thermal strain $\frac{d\varepsilon^{th}}{dt} = \alpha_g \frac{dT}{dt} + (\alpha_l - \alpha_g)\frac{dT_f}{dt}$

α_l **Liquid CTE**

α_g **Glass CTE**

- Shift function

$$\Phi = \exp\left[\frac{H}{R}\left(\frac{1}{T_{ref}} - \frac{x}{T} - \frac{1-x}{T_f}\right)\right]$$

Tool-Narayanaswamy (TN) shift function

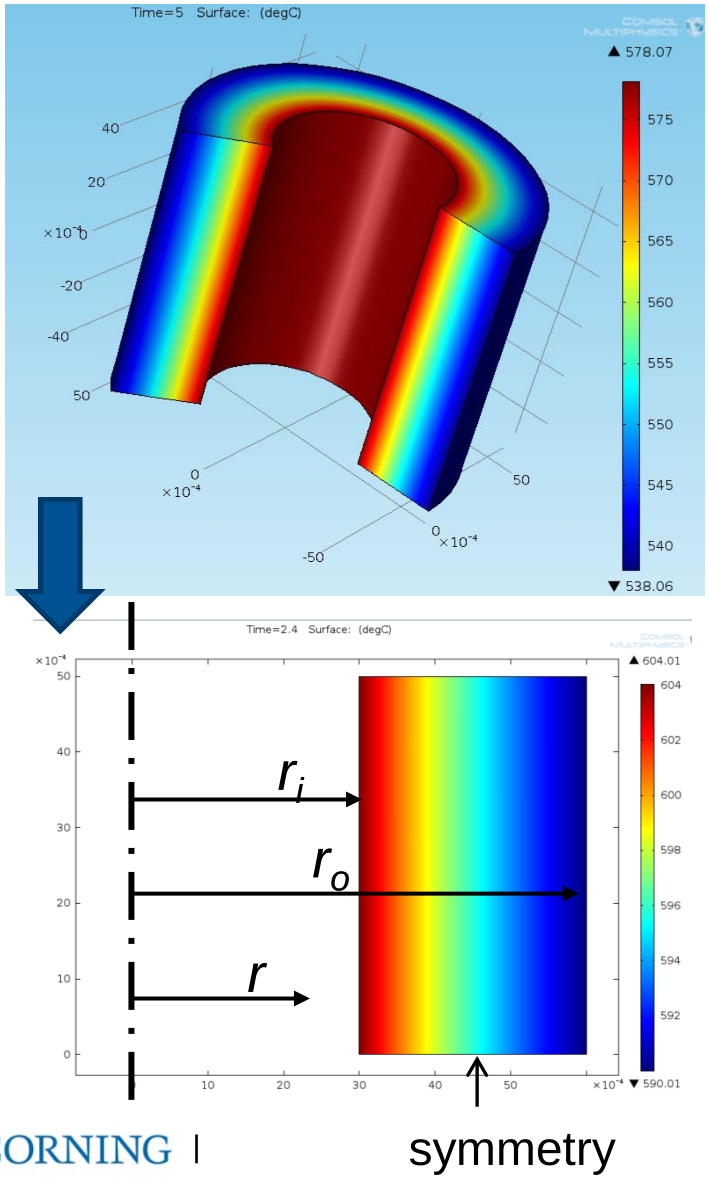
$$\log 10(\Phi) = \frac{-C_1 (T - T_{ref})}{C_2 + (T - T_{ref})}$$

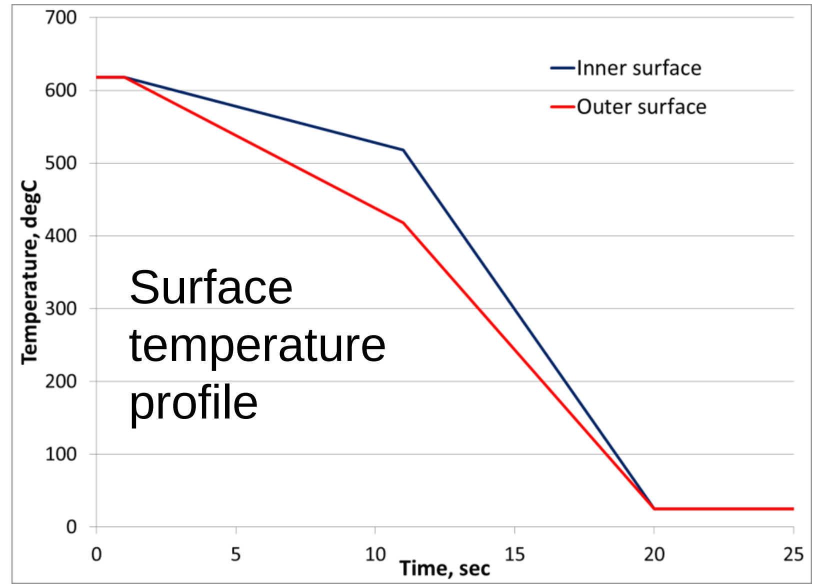
Williams-Landel-Ferry (WLF) shift function

Numerical Example

Temperature distribution in thickness

$$T(r) = T_i + (T_o - T_i)\frac{\ln\left(r/r_i\right)}{\ln\left(r_o/r_i\right)}$$





<u>Simulation process:</u>

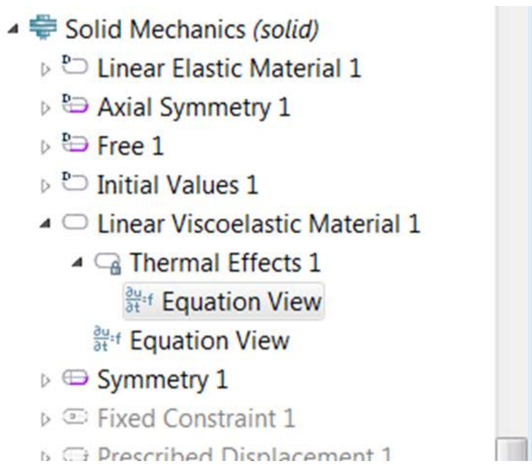
- Glass tube cools from above glass transition to below glass transition;
- Outer surface cools faster than inner surface

COMSOL Implementation

- Physics
  - Solid mechanics module → displacement and stress
    - Linear viscoelastic material added for stress relaxation
  - Domain ODE for fictive temperature
  - Domain ODE for thermal strain

→ Structural relaxation

- Use "Equation View" to update thermal strain and shift function



Name	Expression
solid.Tdiff	solid.T-solid.Tref
solid.eth11	thmeps
solid.eth12	0
solid.eth13	0
solid.eth22	thmeps
solid.eth23	0
solid.eth33	thmeps
solid.alpha11	1

Update thermal strain for stress calculation

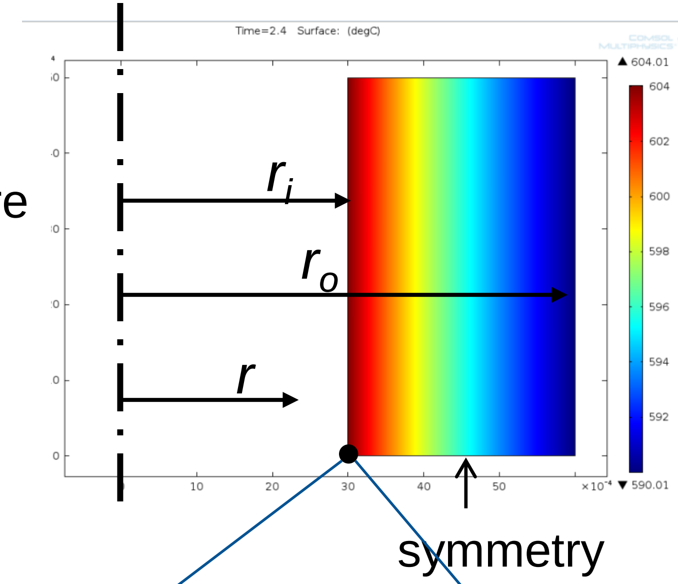
Variables

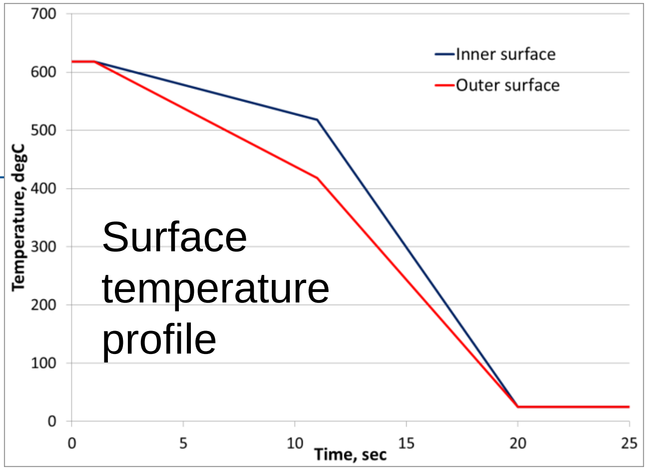
Name	Expression
solid.alpha32	0
solid.alpha13	0
solid.alpha23	0
solid.alpha33	1
solid.shiftWLF	shift_func(myT_var,myTf)
solid.C1wlf	17.44
solid.C2wlf	51.6[K]
solid.Twlf	293.15[K]

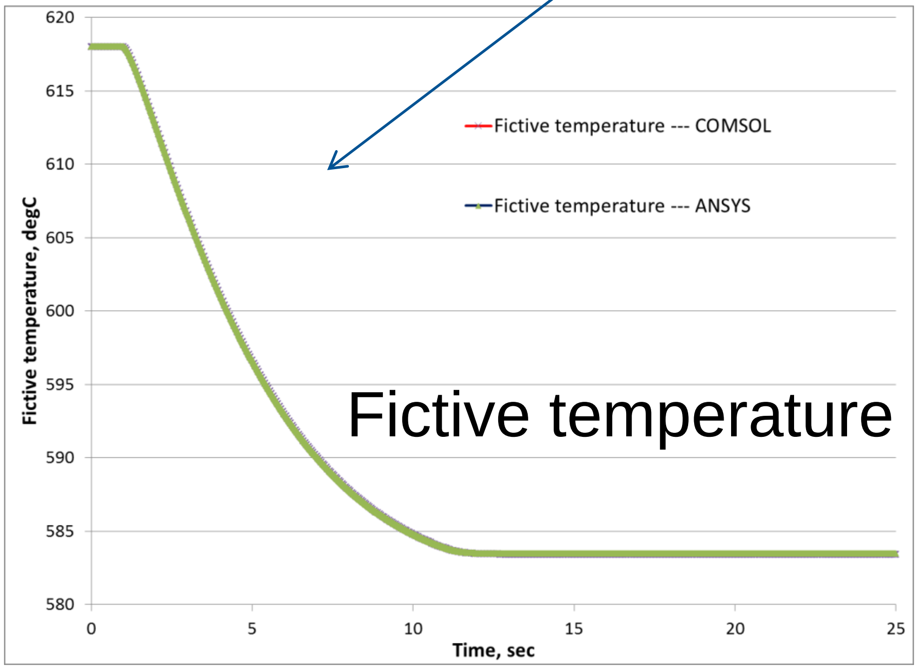
User defined shift function

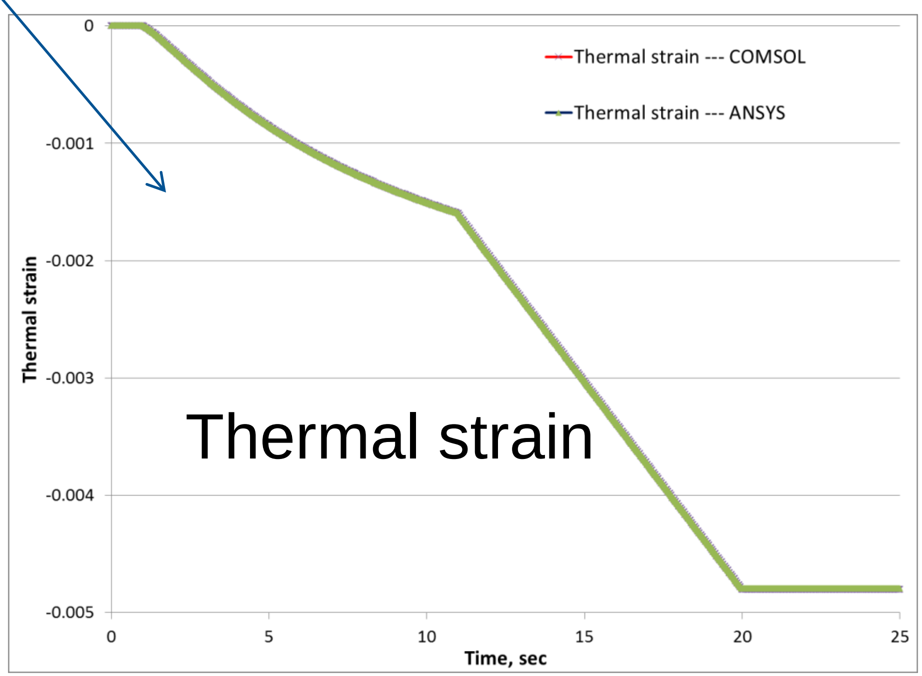
Results

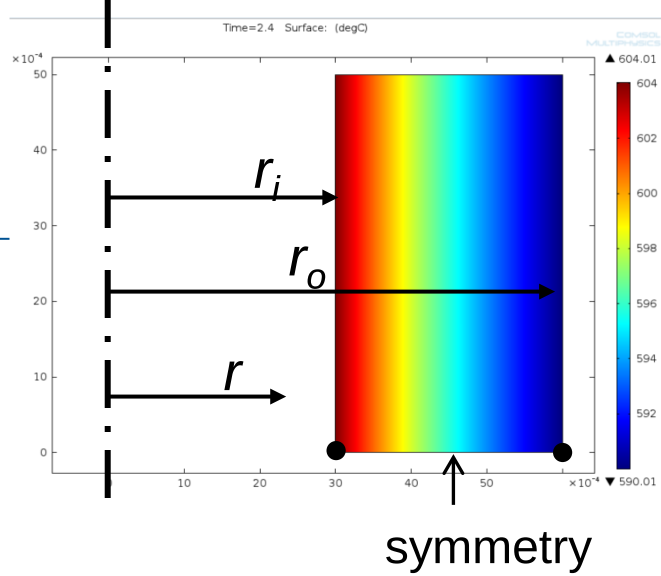
- Fictive temperature and thermal strain solutions
- COMSOL and ANSYS solutions match well

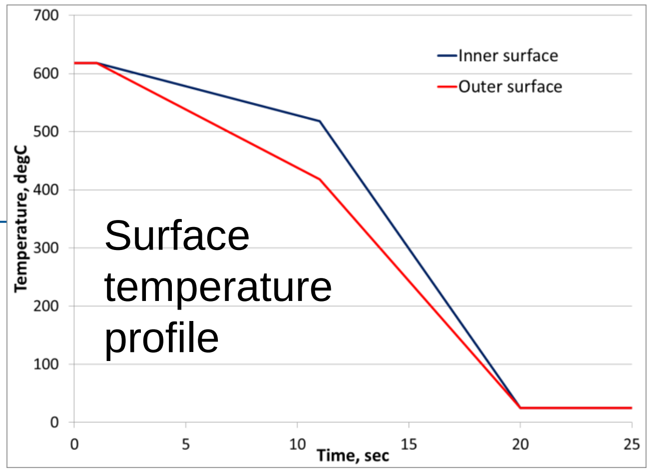






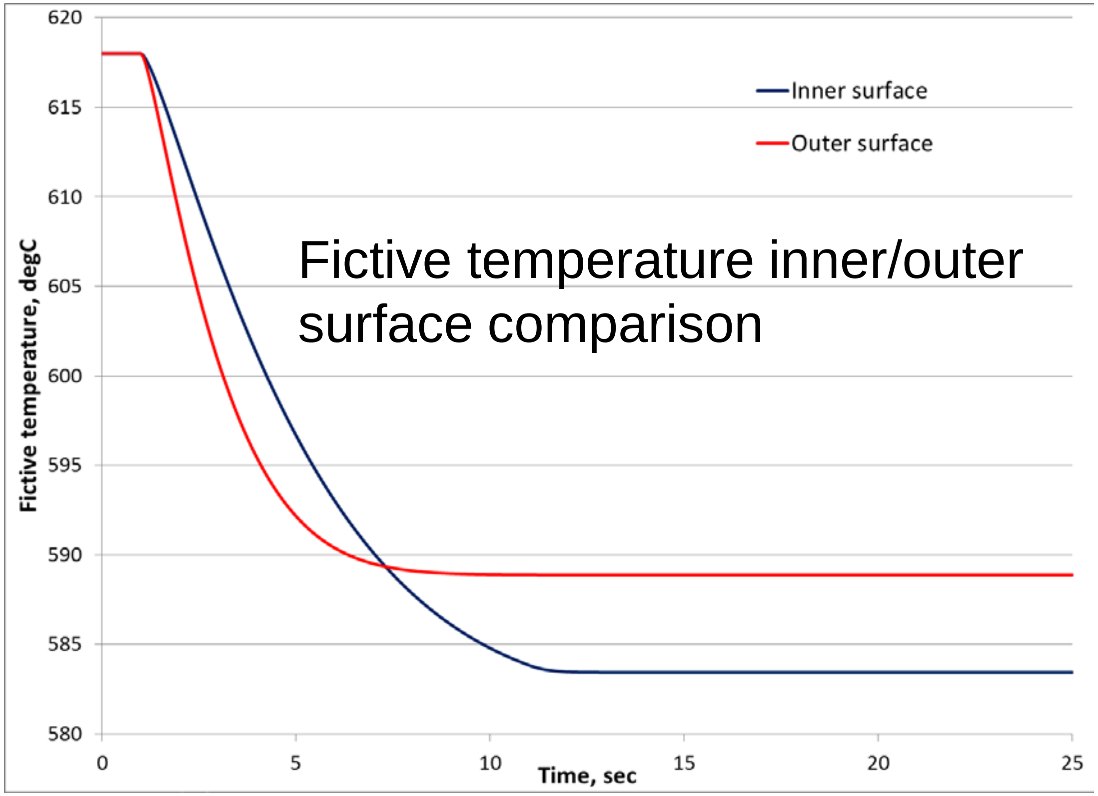


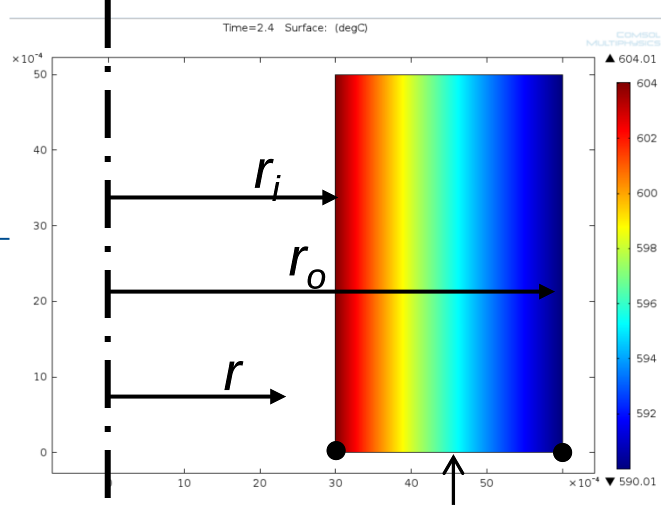




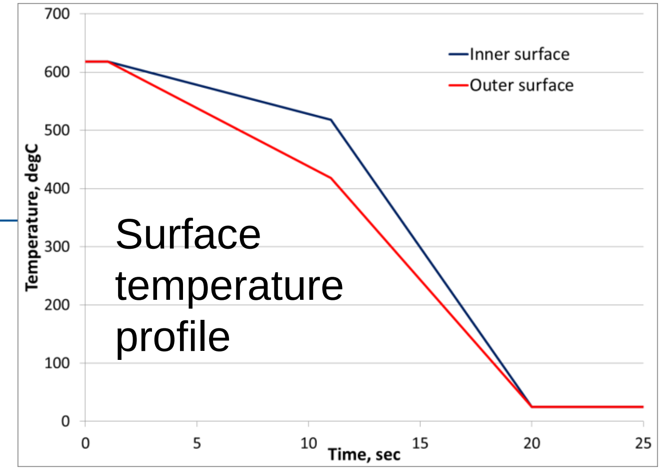
<u>Results</u>

- Outer surface cools faster than inner surface, ends up with higher fictive temperature



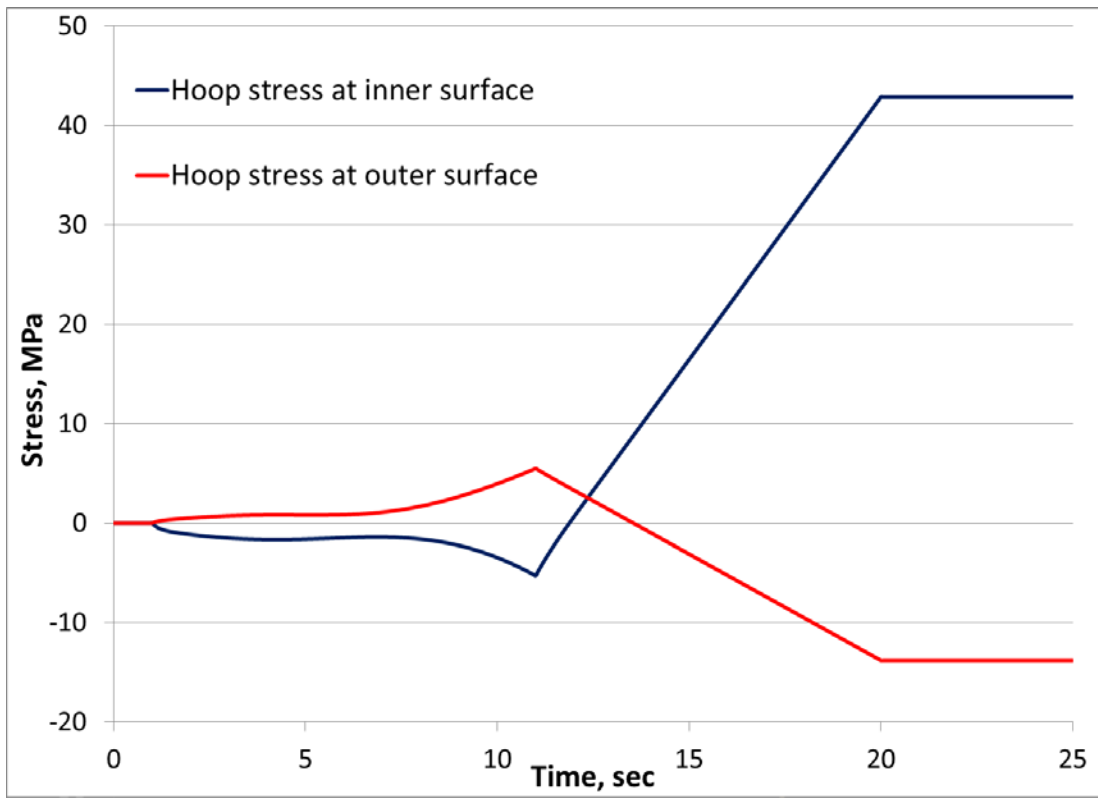


symmetry



Results

- Inner surface is in compression and outer surface in tension at first; they switch sign later.
- Tensile stress at inner surface and compressive stress at outer surface in the end.



Conclusions

- Demonstrated framework of viscoelastic material model implementation in COMSOL 4.3b, including both stress relaxation and structural relaxation, user defined shift function.
- Applicable for various glass viscoelastic simulations with multi-physics: solid mechanics module, heat transfer module, …

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