Parametric Investigation of the Common Geometry Shapes for Added Mass Calculation

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Abstract: This work is aimed to demonstrate how variation of geometries' parameters would affect the fluid loading effect in water using COMSOL *Multiphysics* 4.4 and compared with analytical data. Three-dimensional objects are placed in a water medium. A solid circular cylinder, a solid sphere, and a rectangular cantilever beam are placed in water and the acoustic wave is applied to the objects in water medium. More specifically, background acoustic pressure has been used to simulate an incident plane wave which excites the structures in water. Additionally, the inlet velocity and an outlet pressure gradient are applied for fluid structure interaction investigation. When a structure is placed in water, the interaction between the water and the structure plays an important role in determining the amount of fluid loading mass. The calculated results of added mass of the common geometry shapes match well with the analytical calculations and the discrepancy behavior of added mass amount due to variation of geometries' parameters are established.

Keywords: added mass, acoustics, fluid structure interaction.

1. Introduction

Fluid-Structure Interaction (FSI) occurs due to the interaction of multiple continuum fields. The added mass loading, hydrodynamic mass, is one of the most common FSI phenomenon. Basically the added mass effect is when a structure acts as if it were heavier by an added amount while being overwhelmed by an external continuum. Calculation of added mass has been widely studied in terms of deriving analytical formulas for the flexural and torsional resonant frequencies of common geometry shapes. Sader et al. [1] used the method of Elmer and Dreier to derive explicit analytical formula for the numerical quantity of added mass specifically for the rectangular cantilever beam. Liu et al. [2] designed an experimental setup using a rectangular cantilever beam to estimate the amount of added mass without consideration of any water based assumption. Causin et al. [3] proposed a fluid structure model which simulates the propagation and encounters the added mass effect of incompressible fluid while a circular cylinder is immersed in water. Govardhan et al. [4] focused on vortices, forces and displacements in order to consider the added mass effect on a sphere overwhelmed by water. Singhal et al. [5] derived an analytical model to achieve the different modes and corresponding natural frequencies of a circular cylinder and an experimental setup to validate the model. Lozzo et al. [6] studied the dynamic behavior of a circular cylinder submerged in water while the flexural oscillation was applied.

The primary motivation for this study is to compute the hydrodynamic mass of common geometry shapes such as rectangular cantilever beam, solid sphere and circular cylinder. An extension of this objective is to determine the computed added mass with the variation of geometric parameters so that such correlations may be used for design purposes. In order to see how structures are experiencing the added mass loading effect differently by varying its geometry, it is necessary to investigate the effect of each dimension of its geometry by comparing the numerical values of added mass. In this paper, two hydrodynamics mass equations from the White's [8] are used to estimate the analytical added mass value for each freely placed sphere and circular cylinder when the medium is accelerated at a certain velocity. Moreover, the equations of flexural and torsional resonant frequencies of a rectangular cantilever beam from the Sader et al. [1] have been used to evaluate the analytical amount of added mass when an incident plane wave excites both fluid and structure. The numerical added mass values of the structures obtained from both acoustic domain analysis and fluid domain analysis are compared to each other as well as comparison with the analytical values. Finally, the obtained result shows how varying any of the geometries' parameters would result in different added mass. In COMSOL *Multiphysics* 4.4 module, the Acoustic and Fluid-Structure Interaction physics have been used through a frequency domain analysis and a transient analysis respectively, which will consequently lead to the calculation of added mass.

2. Theory

A rectangular cantilever beam is one of the structures considered for this parametric study. Chu et al. [7] represented the relationship of the natural frequency of a beam in different medium is represented in terms of fluid and vacuum. ρ is density of fluid, ρ_c is density of the beam, h and b are thickness and width of the beam, respectively. Eq. (1) is valid for both viscid and inviscid stationary fluid model at macroscopic level.

$$f_f = f_v\left(1+\frac{\pi\rho b}{4\rho_c h}\right)^{-1/2} \qquad (1)$$

A solid sphere and a circular cylinder are other geometries that have been considered for this parametric study. In the following study R is the radius of the sphere, and the symbol R and L are considered for the radius and the length of the circular cylinder respectively, while both are immersed in water with a density of ρ. The summation of forces on the system will be in terms of mass of the body and hydrodynamic mass (m_h). Based on the potential theory [8], the hydrodynamic mass depends on the shape of a body and the direction of the flow stream which can be calculated by summing the total kinetic energy of the fluid associated with the body and equating it with the kinetic energy of the body where dm is the mass element. Here the V_{rel} and U are velocity of the fluid associated with the body, and velocity of fluid, respectively. Elgabaili [9] have used the calculation of relative velocity (V_{rel}) as an approach for determining the added mass value when both body and the fluid are moving at a certain velocity in which R is the radius, r is the radial direction and θ is the azimuthal angle. Yet, in this parametric study, water is flowing at a certain velocity over the immersed structure. These equations are shown in Eq. (2) to Eq. (5).

$$KE_{fluid} = \int \frac{1}{2} V_{rel}^2 dm = \frac{1}{2} mhU^2 \qquad (2)$$

$$V_{rel}^2 = \frac{U^2}{4}\left(\frac{R}{r}\right)^6 \left(1+3\cos^2\theta\right) \qquad (3)$$

$$m_h = \frac{2}{3}\rho\pi R^3 \text{ for a sphere} \qquad (4)$$

$$m_h = \rho\pi R^2 L \text{ for a cylinder} \qquad (5)$$

3. Use of COMSOL Multiphysics

The 3D pressure acoustics, fluid structure interaction module in COMSOL *Multiphysics* 4.4 have been used through a frequency domain analysis and a transient analysis to demonstrate the added mass loading effect. The geometry shapes are all made up of aluminum 3003-H18 and the dimensions for the given geometries are provided in Table 1. This parametric study is mainly done through varying any of the geometries' parameters that would result in added masses. In order to run the cantilever beam simulation, one of the parameters is varied while the other two parameters are kept fixed. This simulation is repeated until all the parameters are varied while one of the parameters is constant. For the case of a sphere, its radius was changing through a range of values to see how it affects the added mass value. Finally, for a circular cylinder, either the radius was kept or the length was varying or vice versa.

Table 1: Baseline parameters of geometry shapes

Rectangular Cantilever beam		
Type	Symbol	Value
Width Height Length	*b* *a* *L*	0.1 m 0.5 m 4 m
Solid Circular cylinder		
Type	Symbol	Value
Radius Length	*R* *L*	0.5 m 4 m
Solid Sphere		
Type	Symbol	Value
Radius	*R*	0.5 m

3.1 Boundary condition

In the pressure acoustic module, the acoustic wave is applied to the fluid medium which is water, overwhelming the geometries. More specifically, background acoustic pressure field has been used to generate a travelling plane wave which excites the structure in the water. Regarding the boundary conditions in the fluid structure interaction module, an inlet velocity of 1 m/s and an outlet pressure gradient of 0 kPa are applied on the encountering water domain to study the fluid structure interaction, and the water domain outer faces are regarded as hard walls.

The rectangular cantilever beam structure is fixed at one end and it is overwhelmed by a rectangular water environment in both pressure Acoustics and Computational Fluid Dynamics (CFD) studies. On the other hand, for a sphere and a circular cylinder, the water environment is modeled as a sphere and a box in pressure Acoustics and CFD studies, respectively.

3.2 Mesh and solver

In both Acoustics and fluid solid interaction modules, all domains are meshed by Free Tetrahedral elements. In the case of Acoustics module, both of global and submodel are solved in eigenfrequency and frequency domain. In the fluid structure interaction module, the model is solved using time dependent domain analysis. The solver is chosen for MUMPS which is the direct solver.

4. Simulation results and discussion

Based on COMSOL simulations, the eigenfrequencies and kinetic energies are utilized to calculate the added mass in Acoustics and fluid structure interaction modules, respectively. Time harmonic analysis have been performed under two different medium: water and air. These frequency responses obtained from the simulations are represented in Fig.1 (a) and (b). The similar plots were used for each parametric case in order to find the wet (W_{wet}) and dry (W_{dry}) natural frequencies.

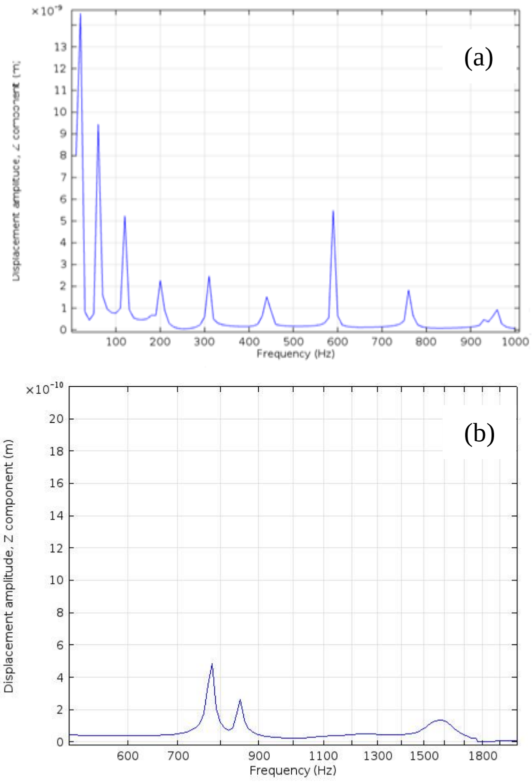

Figure 1: Frequency Response of a) a Solid Sphere , and b) a Solid Circular cylinder.

In the case of fluid structure interaction, the kinetic energy of the structure is obtained over one period of time. The calculated added mass of the common geometry shapes along with the analytical calculations, and the discrepancy behavior of added mass amount due to variation of geometries' parameters are represented in detail. Based on the obtained data, a percentage difference between the added mass values obtained from the two modules are calculated which is defined as "*E(%)* with CFD". In the COMSOL simulation, the total surface displacement result represented for each geometry undergoing this parametric study.

4.1 Rectangular Cantilever Beam

The obtained numerical values of added mass for the cantilever beam using fluid structure interaction module is provided in Table 2 to Table 4. Based on White [8], the integration of relative velocity to find the fluid kinetic energy (KE_{fluid}) can be obtained using body surface integration based on Eq. (2). The positive root of frequency C_n for the cantilever beam is used for each mode. Accordingly, with the use of COMSOL *Multiphysics*, the kinetic energy of the structure is obtained over one period of time (10 seconds) is averaged. Consequently, the calculation has been done for finding the added mass. On the other hand, the calculated added mass using pressure acoustic module is tabulated in Table 5 to Table 7.

$$m_h = 2\rho Lb(a+b) \text{ for a beam} \qquad (6)$$

where *a* and *b* are the height and the width of the beam.

$$m_{add} = \rho_{add} \times V \qquad (7)$$

where *V* is the volume of the beam and ρ_{add} is the added density which is represented in Eq. (8).

$$\rho_{add} = \frac{N^2 C_n^4 EI}{W_{wet}^2 A L^4} \qquad (8)$$

where *N* is the ratio between W_{wet} and W_{dry} and *E* is the young’s modulus of the beam.

Table 2: Fixed Length "*L*" of the cantilever beam using fluid structure interaction module

Height *a* (m)	Width *b* (m)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with Analytical	*KE* Fluid (KJ)
0.5	0.1	546.00	542.68	11.55	271.34
0.6	0.2	1310.40	1401.34	9.48	700.67
0.7	0.3	2293.20	2270.51	5.71	1135.25
0.8	0.4	3494.40	3440.19	21.13	1720.09
0.9	0.5	4914.00	4789.53	14.47	2394.76
1	0.6	6552.00	6497.11	15.41	3248.55
0.4	0.7	3057.60	3009.24	51.15	1504.62
0.3	0.8	2620.80	2600.15	63.03	1300.07
0.2	0.9	1965.60	1901.79	75.98	950.89
0.1	1	13155.00	13126.96	49.17	6563.48

Table 3: Fixed Height "*a*" of the cantilever beam using fluid structure interaction module

Length *L* (m)	Width *b* (m)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with Analytical	*KE* Fluid (KJ)
4	0.1	546.00	542.68	11.55	271.34
3	0.2	819.00	950.96	13.21	475.48
2	0.3	819.00	809.79	15.64	404.89
1	0.4	546.00	542.53	24.65	271.26
5	0.5	3412.50	3398.87	32.02	1699.43
6	0.6	4914.00	4876.25	38.43	2438.13
7	0.7	6688.50	6592.95	43.93	3296.47
8	0.8	8736.00	8688.19	47.78	4344.09
9	0.9	11056.50	10997.58	51.51	5498.79
10	1	13650.00	13255.37	55.82	6627.68

Table 4: Fixed Width "*b*" of the cantilever beamusing fluid structure interaction module

Length *L* (m)	Height *a* (m)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with Analytical	*KE* Fluid (KJ)
4	0.5	546.00	542.68	11.55	271.34
3	0.4	327.60	321.84	6.78	160.92
2	0.3	163.80	171.51	7.19	85.75
1	0.2	54.60	53.45	10.92	26.73
5	0.1	136.50	223.18	11.60	111.59
6	0.6	982.80	966.68	13.11	483.34
7	0.7	1337.7	1333.92	16.04	666.96
8	0.8	1747.20	1736.35	17.07	868.17
9	0.9	2211.30	2188.86	17.76	1094.43
10	1	2730.00	2692.15	18.28	1346.07

According to the obtained data, as the cross section of the rectangular cantilever beam increases the numerical value of added mass is increasing. When the length is kept constant, the added mass value increases as the cross section increases in dimension, and it gets lower while the cross section decreases. The obtained percentage differences are relatively high which shows the need for further investigation on parametric analysis.

In the case of pressure Acoustics simulation, the wet and dry natural frequencies are obtained from the frequency plots. These natural frequencies are used in the relation with which Sader et al. [1] have suggested for the added mass. In order to find the natural frequency of the submerged cantilever beam, an incident plane wave has been applied through a range of different frequencies, in both air and water mediums, such as 10 Hz and 60 Hz as shown in Fig. 2 (a) and (b) respectively.

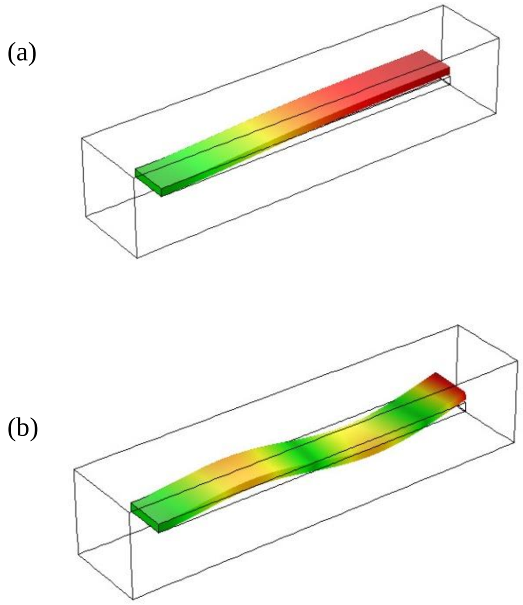

Figure 2: Surface Total Displacement of a rectangular cantilever beam in Frequency Domain of a) 10 Hz, and b) 60Hz.

Table 5: Fixed Length "*L*" of the cantilever beam using pressure acoustic module

Height *a* (m)	Width *b* (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
0.5	0.1	155.07	159.47	546.00	543.29	0.13
0.6	0.2	182.76	191.33	1310.40	1296.17	0.17
0.7	0.3	210.61	223.22	2293.20	2276.13	0.25
0.8	0.4	238.06	254.61	3494.40	3457.14	0.49
0.9	0.5	266.16	286.64	4914.00	4823.99	0.72
1	0.6	294.48	318.89	6552.00	6502.91	0.18
0.4	0.7	103.97	127.49	3057.60	3016.73	0.25
0.3	0.8	71.96	95.67	2620.80	2605.47	0.21
0.2	0.9	32.39	63.78	1965.60	1905.21	0.18
0.1	1	16.19	31.89	13155.00	1058.68	0.17

Table 6: Fixed Height "*a*" of the cantilever beam using pressure acoustic module

Length *L* (m)	Width *b* (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
4	0.1	155.07	159.47	542.68	543.29	0.13
3	0.2	268.41	283.43	950.96	806.93	0.25
2	0.3	588.96	637.77	809.79	812.86	0.35
1	0.4	2300.17	2551.18	542.53	544.05	0.28
5	0.5	89.92	102.04	3398.87	3403.05	0.12
6	0.6	61.09	70.86	4876.25	4890.32	0.29
7	0.7	43.95	52.06	6592.95	6621.07	0.42
8	0.8	32.97	39.89	8688.19	8793.21	1.12
9	0.9	25.56	31.49	10997.58	11041.24	0.39
10	1	20.31	25.49	13255.37	13419.05	1.22

Table 7: Fixed Width "*b*" of the cantilever beam using pressure acoustic module

Length *L* (m)	Height *a* (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
4	0.5	155.07	159.47	546.00	543.29	0.13
3	0.4	218.95	226.69	327.60	323.65	0.61
2	0.3	365.53	382.66	163.80	162.01	0.64
1	0.2	954.36	1020.69	54.60	54.02	1.05
5	0.1	17.98	20.41	136.50	136.14	0.25
6	0.6	83.07	85.04	982.80	969.01	0.24
7	0.7	71.42	72.78	1337.7	1334.19	0.76
8	0.8	62.61	63.73	1747.20	1738.44	0.12
9	0.9	55.81	56.69	2211.30	2191.29	0.11
10	1	50.29	51.01	2730.00	2697.66	0.21

In this case, as it is provided in the obtained data shown in Table 7, the wet natural frequencies are lower compared to the dry frequencies as it was expected due to the added mass effect. It has been found that for the constant length of the beam, the cross section and added mass are experiencing a linearly proportional relationship, but when the constant parameter is the height, the relationship is no longer proportionally linear. The added mass increases as the cross section increases, but at length of 1 m and width of 0.4 m added mass value drops. After this combination of parameters, there is linear relationship between the rectangular cross section and added mass. Finally, when the width is the constant, there is also a linearly increasing relationship between the numerical value of cross section and added mass. The percentage difference of the added mass obtained using acoustic module and fluid structure interaction module is small, meaning that the results obtained from two different methods are close to each other.

4.2 Solid Sphere

The added mass values for the solid sphere based on the fluid structure interaction simulation are provided in Table 8, and the calculated added mass using pressure Acoustics module is tabulated in Table 9. Based on White [8], the integration of relative velocity to find the fluid kinetic energy (KE_{Fluid}) can be obtained using body surface integration. Accordingly, with the use of COMSOL *Multiphysics*, the kinetic energy of the structure obtained over one period of time (10 seconds) is averaged to find the added mass based on Eq. (2) and Eq. (4). The solid sphere's total surface displacement for both acoustic and CFD analysis at the first parametric sweep are shown in Fig.3 (a) and (b).

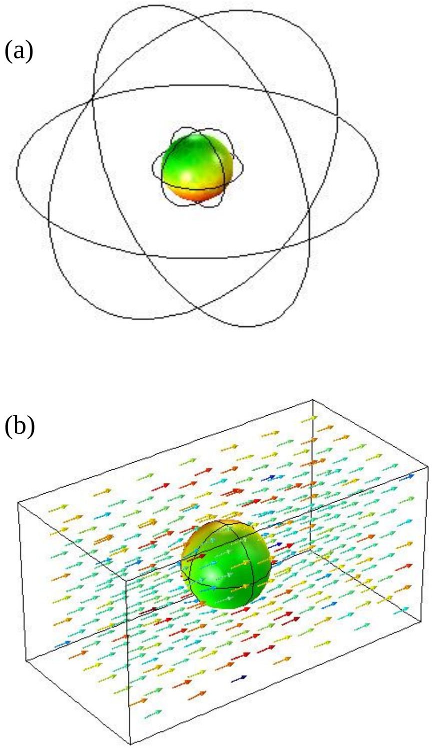

Figure 3: Surface Total Displacement for a solid sphere in a) Acoustic Domain, and b) Fluid Structure Interaction.

As it is shown in Table 9, the wet natural frequencies are lower compared to the dry frequencies. On the other hand, the added mass increases as the radius of sphere increases and vice versa. The low percentage difference between analytical and obtained added mass values shows that the obtained results are congruent to each other.

Table 8: Varying radius "*R*" of the solid sphere using fluid structure interaction module

R (m)	Body Mass (Kg)	Added Mass (Kg)	E(%) with Analytical	*KE* Fluid (KJ)
0.5	1429.42	263.38	0.61	131.69
0.6	2470.04	456.23	0.85	228.11
0.7	3922.34	721.75	0.47	360.87
0.8	5854.92	1078.22	0.55	539.11
0.9	8336.40	1528.64	0.12	764.32
1	11435.39	2102.35	0.38	1051.17
0.4	731.86	135.26	0.91	67.63
0.3	308.75	56.68	0.24	28.34
0.2	91.48	16.88	0.79	8.44
0.1	11.44	2.09	0.19	1.05

Table 9: Varying radius "R" of the solid sphere using pressure acoustic module

R (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
0.5	37.5	75.7	1429.42	266.72	1.25
0.6	46.4	74.8	2470.04	461.53	1.15
0.7	23.4	40.1	3922.34	730.27	1.17
0.8	20.2	36.8	5854.92	1100.81	2.05
0.9	19.3	36.2	8336.40	1550.85	1.43
1	10.1	20.5	11435.39	2145.92	2.03
0.4	104.1	106.3	731.86	138.18	2.11
0.3	112.5	118.7	308.75	57.61	1.61
0.2	115.2	122.4	91.48	17.32	2.54
0.1	100.4	107.5	11.44	2.12	1.42

The obtained data shown in the Table 9 reveals that the wet natural frequencies are lower compared to the dry frequencies. The same relationship along with varying the radius to its corresponding added mass has also come to the same conclusion as CFD analysis. Yet there is a relatively high percentage difference between the added mass values obtained using both modules. This low percentage difference between the Acoustics and fluid structure interaction analysis shows the closeness of the modules for the same case study.

4.3 Solid Circular Cylinder

The numerical added mass values for the solid circular cylinder using the fluid structure interaction module is provided in Table 10 and Table 11. Once again based on White [8], the integration of relative velocity to find the fluid kinetic energy (KE_{Fluid}) has been obtained using body surface integration. Accordingly, with the use of COMSOL *Multiphysics*, the kinetic energy of the structure obtained over one period of time (10 seconds) is averaged. Consequently, the calculation has been done for finding the added mass based on Eq. (2) and Eq. (5). On the other hand, the calculated added mass using pressure Acoustics module is tabulated in Table 12 and Table 13. The solid circular cylinder's total surface displacement for both acoustic and CFD analysis at the first parametric sweep are shown in Fig.4 (a) and (b).

According to the data obtained for circular cylinder's parametric sweep, there is an increase in the added mass value as either radius or length increases. The low percentage difference with the analytical values shows that the results are agreeable to each other.

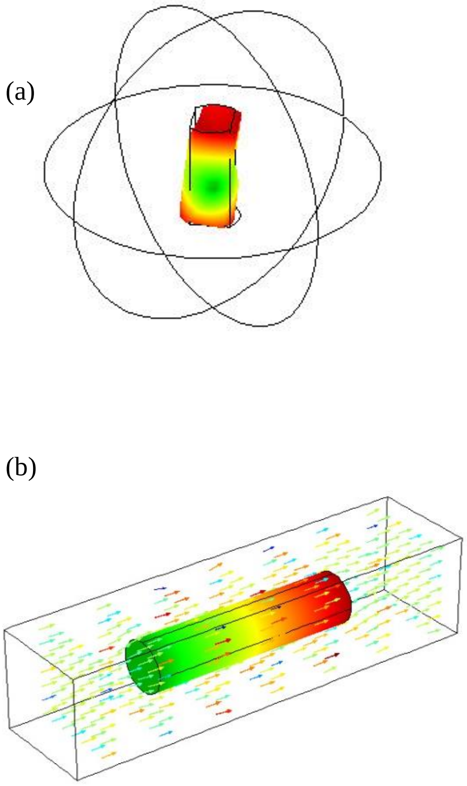

Figure 4: Surface Total Displacement for a solid circular cylinder in a) Acoustic Domain , and b) Fluid Structure Interaction.

Judging from the data obtained using Acoustics module, there is also an increase in the added mass value as either radius or length increases as we observed from the CFD simulation analysis. The low percentage difference with the CFD simulation has been obtained.

Table 10: Fixed Length "*L*" of the cylinder using fluid structure interaction module

R (m)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with Analytical	*KE* Fluid (KJ)
0.5	8576.55	3169.55	0.89	1584.77
0.6	12350.23	4560.98	0.82	2280.49
0.7	16810.03	6200.62	0.74	3100.31
0.8	21955.96	8076.25	0.42	4038.12
0.9	27788.02	10230.67	0.51	5115.33
1	34306.19	12661.87	0.76	6330.93
0.4	5488.99	2022.08	0.57	1011.04
0.3	3087.56	1140.47	0.84	570.23
0.2	1372.25	503.91	0.25	251.95
0.1	343.06	126.02	0.28	63.01

Table 11: Fixed Radius "*R*" of the cylinder using fluid structure interaction module

L (m)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with Analytical	*KE* Fluid (KJ)
4	8576.55	3169.55	0.89	1584.77
5	10720.68	3948.19	0.54	1974.09
6	12864.82	4743.48	0.66	2371.74
7	15008.95	5524.72	0.49	2762.36
8	17153.09	6327.79	0.71	3163.89
9	19297.23	7108.16	0.56	3554.08
10	21441.37	7912.09	0.74	3956.04
3	6432.41	2365.14	0.38	1182.57
2	4288.27	1585.24	0.92	792.62
1	2144.14	788.61	0.41	394.31

Table 12: Fixed Length "*L*" of the cylinder using pressure acoustic module

R (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
0.5	34.6	47.5	8576.55	3179.72	0.32
0.6	36.6	50.2	12350.23	4578.84	0.41
0.7	34.2	46.8	16810.03	6219.12	0.29
0.8	31.18	42.8	21955.96	8189.91	1.38
0.9	28.4	39	27788.02	10375.92	1.39
1	25.2	34.6	34306.19	12835.96	1.36
0.4	33.3	45.7	5488.99	2043.38	1.04
0.3	22.5	30.9	3087.56	1155.27	1.28
0.2	16.9	23.2	1372.25	513.94	1.95
0.1	7.8	10.7	343.06	127.68	1.31

Table 13: Fixed Radius "*R*" of the cylinder using pressure acoustic module

L (m)	W_{wet} (rad/s)	W_{dry} (rad/s)	Body Mass (Kg)	Added Mass (Kg)	*E(%)* with CFD
4	34.6	47.4	8576.55	3172.82	0.11
5	24.6	33.6	10720.68	3926.78	0.54
6	16.2	22.3	12864.82	4784.16	0.85
7	12.5	17.2	15008.95	5621.55	1.72
8	10.1	13.9	17153.09	6412.12	1.32
9	8.3	11.4	19297.23	7161.72	0.75
10	4.9	6.8	21441.37	7989.85	0.97
3	44.3	60.9	6432.41	2406.74	1.73
2	58.6	80.4	4288.27	1597.95	0.79
1	72.8	100.1	2144.14	803.88	1.89

5. Conclusion

This parametric study was studied numerically to investigate the variation of geometries' parameters and the corresponding behavior of the fluid loading effect in water using COMSOL *Multiphysics* 4.4. The numerical added mass values of the structures obtained from both Acoustics domain and fluid domain analyses are compared to each other as well as with the analytical values. These simulated and calculated results of added mass of the common geometry shapes match well with the analytical calculations and the discrepancy behavior of added mass amount due to variation of geometries' parameters are established for the common geometry shapes. For the rectangular cantilever beam, it has been found that increase in length and the cross section will result in the increase of added mass value. In the case of sphere, the obtained result shows the relation in which increase in the added mass value happens as the radius increases. Finally, the parametric study on the circular cylinder revealed that as either radius or length is increasing the added mass will be higher. The analytical formulas suggested by White [8] works out the best for the circular cylinder and sphere, yet the natural frequency based formula recommended by Chu et al. [7] works the best for rectangular cantilever beam. The motivation for this study was to compute the hydrodynamic mass of common geometry shapes such as rectangular cantilever beam, solid sphere and circular cylinder. An extension of this objective was to determine the computed added mass with the variation of geometric parameters so that such correlations may be used for design purposes. For future works, it is highly recommended to do more wider parametric studies in order to obtain much more accurate added mass equations for the common geometry shapes.

6. References

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