Bloch Waves in an Infinite, Periodically-perforated Sheet

W. Maysenhölder[1]

[1]Fraunhofer Institute for Building Physics, Stuttgart, Germany

Abstract

Bloch waves in infinite periodic structures - much in vogue in the present metamaterial age [1, 2] - can be conveniently studied by COMSOL Multiphysics®. This is demonstrated by a simple, yet rich two-dimensional example: a perforated sheet with square symmetry (Figure 1). Its implementation in COMSOL is quite simple [3], the essential feature being "Solid Mechanics > Connections > Periodic Condition > Periodicity Settings > Floquet Periodicity" in both x- and y-directions. Instead of plane waves in homogeneous media, one has to deal with their generalizations: the Bloch waves. The frequencies of such Bloch waves are obtained from the solutions of an eigenvalue problem with prescribed wave vector resulting in dispersion relations which make up the band structure (Figure 2). The characteristic features of such (running or standing) Bloch waves can be examined with the COMSOL plotting and animation tools. E.g., it is apparent in Figure 3 that the deformation of the top and bottom sides of the unit cells is not exactly sinusoidal as it would be the case with a plane wave.

In addition, evaluation and visualization of kinetic and potential energy densities as well as of the structure-borne sound intensity vector - second-order quantities which are less frequently addressed - help to analyze energy distributions and energy transport associated with wave propagation (Figure 4). Sometimes the group velocity as deduced from the dispersion is negative, i.e., energy is - on average - propagating in the opposite direction compared to the propagation of the phase. Figure 4 shows such an example. In non-symmetry directions wave vector and average intensity are usually neither parallel nor antiparallel. These phenomena can be confirmed and quantified by averaging the intensity over a unit cell. At some frequencies and directions, vortices emerge in the intensity field: an intriguing feature which is certainly not yet completely explored (see, e.g., [4] for a discussion of intensity vortices in finite thin plates).

Thanks to two theorems related to the energy of elastic Bloch waves [5], the numerical accuracy of some unit-cell averages may be checked. Moreover, at low frequencies analytical solutions are available for comparison [6-9]. They also enable an exact homogenization to an anisotropic effective medium [10]. In this regime, the properties of a Bloch wave are fairly easily understood on the basis of plane waves in a homogeneous anisotropic medium. However, for higher frequencies where analytical results are rarely available one has to resort to numerical methods and physical interpretations become more complex. COMSOL offers a valuable and powerful tool for investigating periodic

structures in a wide range of frequencies. In particular, detailed consideration of the energy aspect might open new perspectives, also and especially for metamaterial research.

Reference

[1] P.A. Deymier, Ed, Acoustic Metamaterials and Phononic Crystals, Berlin, Heidelberg: Springer (2013)
[2] R.V. Craster and S. Guenneau, Eds, Acoustic Metamaterials: Negative Refraction, Imaging, Lensing and Cloaking, Dordrecht: Springer (2013)
[3] N. Elabbasi, Modeling Phononic Band Gap Materials and Structures, [Online] Available: https://www.comsol.com/blogs/modeling-phononic-band-gap-materials-and-structures/. Accessed on: Jul. 19 2016
[4] N. Tanaka et al., “Vorticity characteristics of the vibrational intensity field in an actively controlled thin plate,” Journal of the Acoustical Society of America, vol. 99, no. 2, pp. 942–953 (1996)
[5] W. Maysenhölder, “Proof of two theorems related to the energy of acoustic Bloch waves in periodically inhomogeneous media,” ACUSTICA, vol. 78, pp. 246–249, (1993)
[6] W. Maysenhölder and R. Haberkern, “Computational investigation of energy propagation in periodic and layered continua,” in Proceedings NOVEM 2000, Lyon / France (CD-ROM), pp. 1–11 (2000)
[7] W. Maysenhölder, Énergie vibratoire: Bases pour le calcul des densités d’énergie et des intensités. Traduction française par Nicolas JOLY (Laboratoire d’Acoustique de l’Université du Maine) de l’ouvrage ”Körperschallenergie: Grundlagen zur Berechnung von Energiedichten und Intensitäten” Waldemar Maysenhölder, 1994 ISBN 3-7776-0607-3: édition originale en langue allemande : Hirzel (2010)
[8] W. Maysenhölder, Körperschallenergie: Grundlagen zur Berechnung von Energiedichten und Intensitäten. Stuttgart: Hirzel (1994)
[9] W. Maysenhölder, “Körperschallintensitäten und -energiedichten in periodischen Medien,” IBP-Mitteilung, vol. 16, no. 187, pp. 1–2 (1989)
[10] A. N. Norris, “On the acoustic determination of the elastic moduli of anisotropic solids and acoustic conditions for the existence of symmetry planes,” Quarterly Journal of Mechanics and Applied Mathematics, vol. 42, no. 3, pp. 413–426 (1989)

Figures used in the abstract

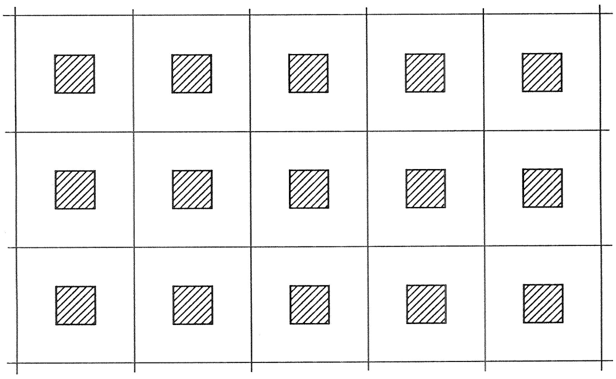

Figure 1: Infinite two-dimensional periodic solid sheet with square holes: "perforated sheet".

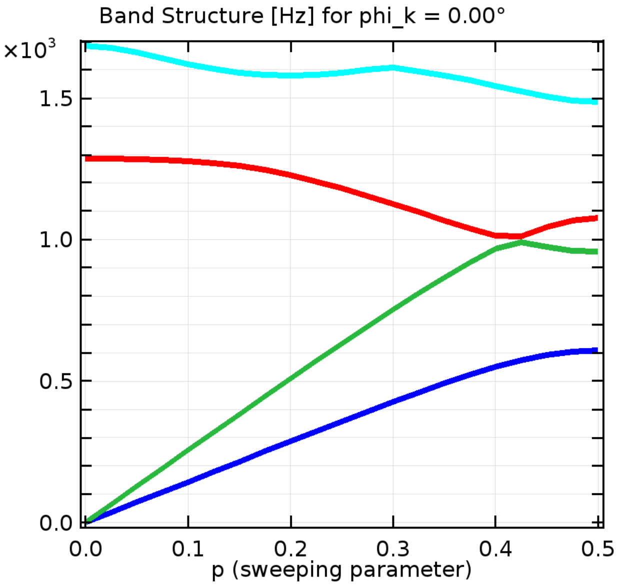

Figure 2: Frequency-wavenumber diagram for Bloch waves propagating along the x-direction (Phi=0°). p denotes normalized wave number.

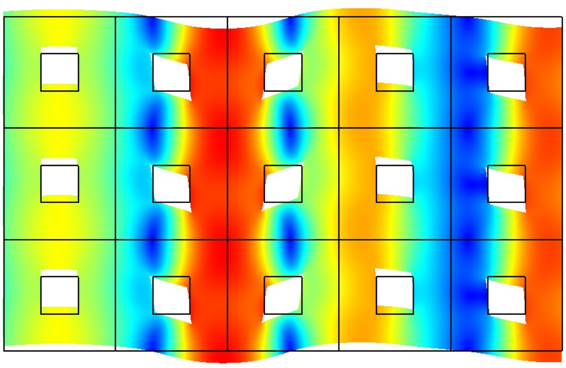

Figure 3: Bloch wave of shear type (see blue branch in Figure 2 at p = 0.35 and 491 Hz; Color: displacement).

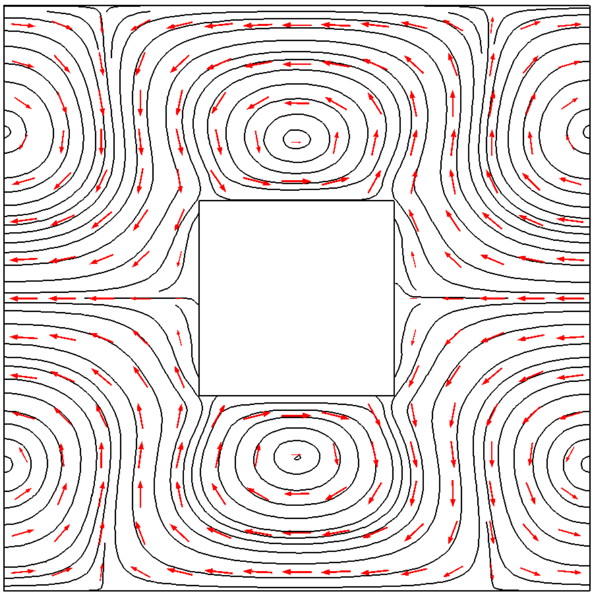

Figure 4: Structure-borne sound intensity (streamlines and arrows) of a Bloch wave (see light-blue branch in Figure 2 at p = 0.15 and 1589 Hz).