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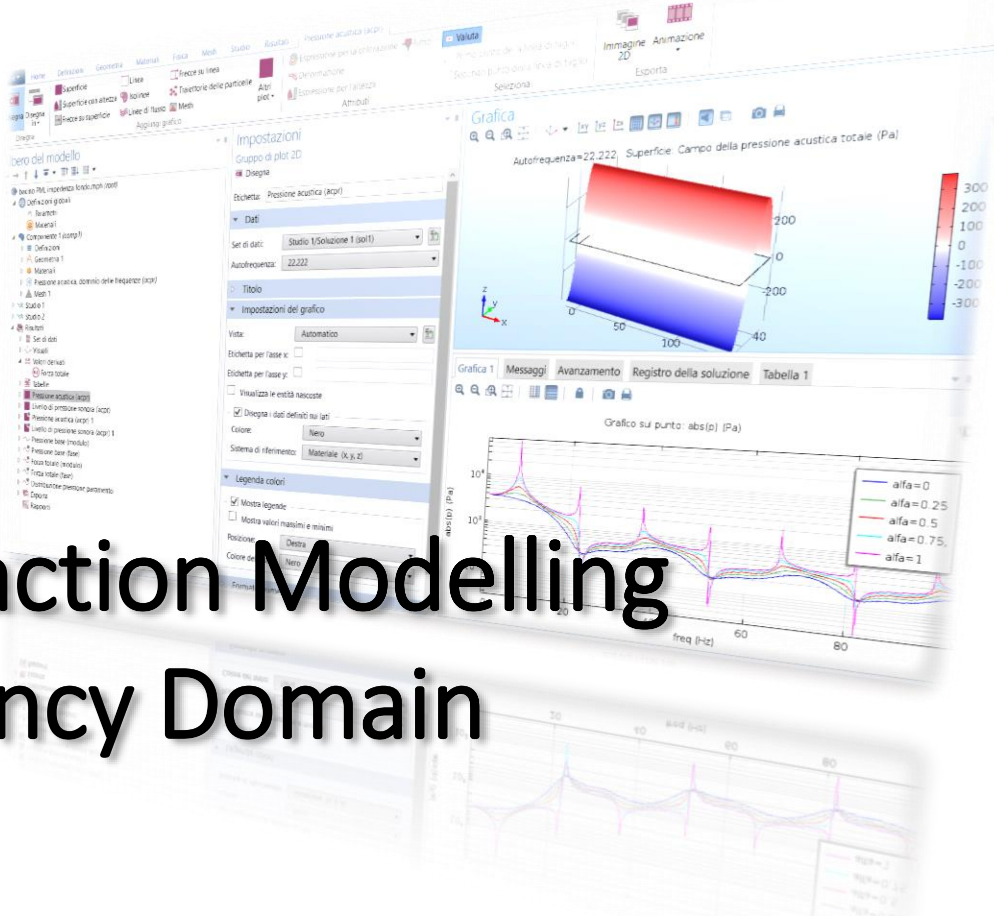

Acoustic Fluid-Structure Interaction Modelling of Gravity Dams in the Frequency Domain

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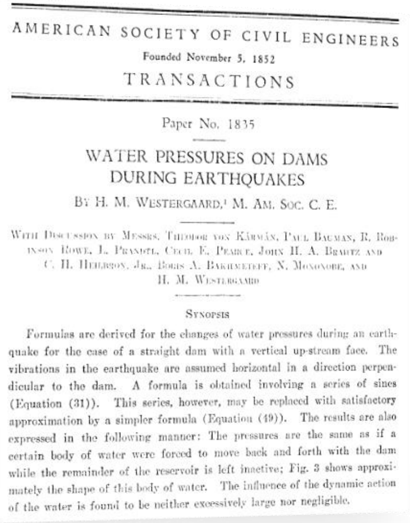
Adoption of the acoustic formulation:

Previous works:

H. M. Westergaard (1933) - A. K. Chopra (1967)

> *«… Since the involved motions are small, the relatively simple equations may be used, that apply to sound in liquids. … »*



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WATER PRESSURES ON DAMS
DURING EARTHQUAKES

BY H. M. WESTERGAARD,[1] M. AM. SOC. C. E.

WITH DISCUSSION BY MESSRS. THEODOR VON KÁRMÁN, PAUL BAUMAN, R. ROBINSON ROWE, L. PRANDTL, CECIL E. PEARCE, JOHN H. A. BRAHTZ AND C. H. HEILBRON, JR., BORIS A. BAKHMETEFF, N. MONONOBE, AND H. M. WESTERGAARD

SYNOPSIS

Formulas are derived for the changes of water pressures during an earthquake for the case of a straight dam with a vertical up-stream face. The vibrations in the earthquake are assumed horizontal in a direction perpendicular to the dam. A formula is obtained involving a series of sines (Equation (31)). This series, however, may be replaced with satisfactory approximation by a simpler formula (Equation (49)). The results are also expressed in the following manner: The pressures are the same as if a certain body of water were forced to move back and forth with the dam while the remainder of the reservoir is left inactive; Fig. 3 shows approximately the shape of this body of water. The influence of the dynamic action of the water is found to be neither excessively large nor negligible.

Author's assumptions:

$$\nabla^2 p = \frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2}$$

- Null global flow
- Rigid dam
- Infinite-length reservoir
- Small displacements of the fluid particles
- Bidimensional problem (2D)

Westergaard added mass method:

Fourier series solution for particle displacement:

$$\xi = -\frac{\alpha g T^2}{\pi^2}\cos\left(\frac{2\pi t}{T}\right)\sum_{1,3,5\,\ldots}^{n}\frac{1}{n}e^{-q_n}\sin\left(\frac{n\pi y}{2h}\right)$$

$$\eta = \frac{\alpha g T^2}{\pi^2}\cos\left(\frac{2\pi t}{T}\right)\sum_{1,3,5\,\ldots}^{n}\frac{1}{n\boldsymbol{c_n}}e^{-q_n}\cos\left(\frac{n\pi y}{2h}\right)$$

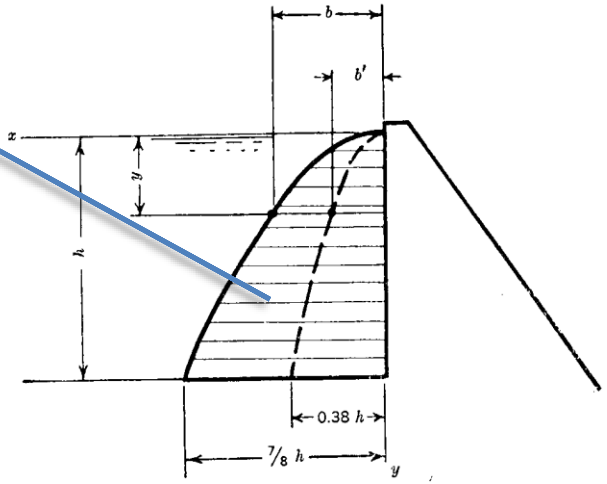
$$c_n = \sqrt{1-\frac{16wh^2}{n^2 gkT^2}}$$

Reservoir period(s):

$$T_n = \frac{4h}{c\cdot n}$$

Added Water Mass:

$$b(y) = \frac{7}{8}\sqrt{hy}$$



Fluid Pressure

p

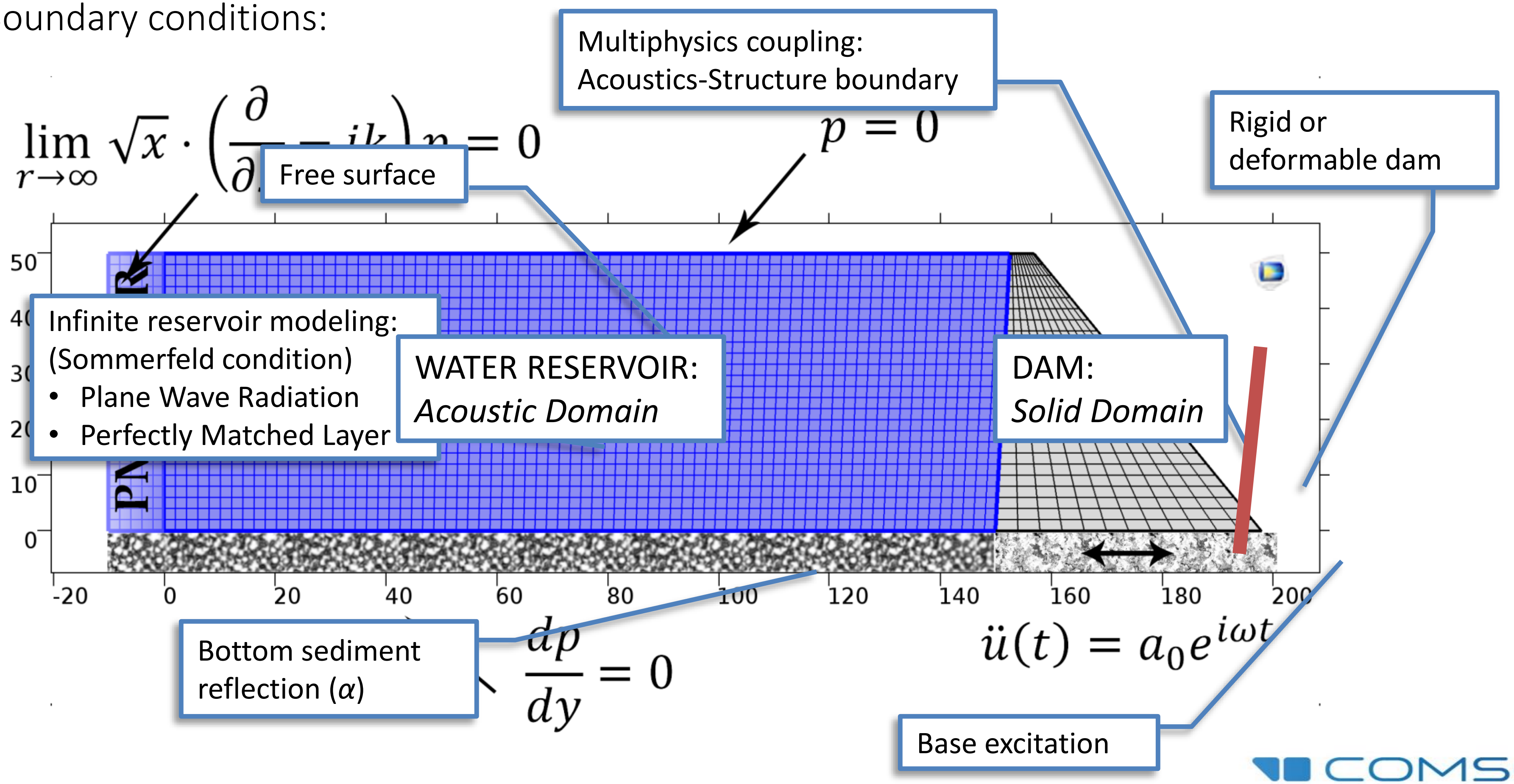
F=ma

Inertial Forces

The COMSOL® model:

- Pressure acoustics interface – frequency domain Structural mechanics

Boundary conditions:



Multiphysics coupling:
Acoustics-Structure boundary

Rigid or
deformable dam

$$\lim_{r\to\infty}\sqrt{x}\cdot\left(\frac{\partial}{\partial}-ik\right)p=0$$

Free surface

$$p=0$$

Infinite reservoir modeling:
(Sommerfeld condition)

- Plane Wave Radiation
- Perfectly Matched Layer

WATER RESERVOIR:
Acoustic Domain

DAM:
Solid Domain

Bottom sediment
reflection (α)

$$\frac{dp}{dy}=0$$

$$\ddot{u}(t)=a_0e^{i\omega t}$$

Base excitation

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Modeling the Infinte

Two approaches:

- **Plane wave radiation (PWR)**: Robin boundary condition, second order

$$-\boldsymbol{n}\cdot\left(-\frac{1}{\rho_c}\left(\nabla p_t-\cancel{\boldsymbol{q}_{\boldsymbol{d}}}\right)\right)+i\frac{k}{\rho_c}p+\frac{i}{2k\rho_c}\Delta_T p=\cancelto{0}{Q_i}$$

No monopole or dipole sources

- **Perfectly matched layer (PML)**: Complex coordinate transformation
  - Rational scaling:

$$f_r(\xi)=s\lambda\xi\left(\frac{1}{3p(1-\xi)+4}-\frac{i}{3p(1-\xi)}\right)$$

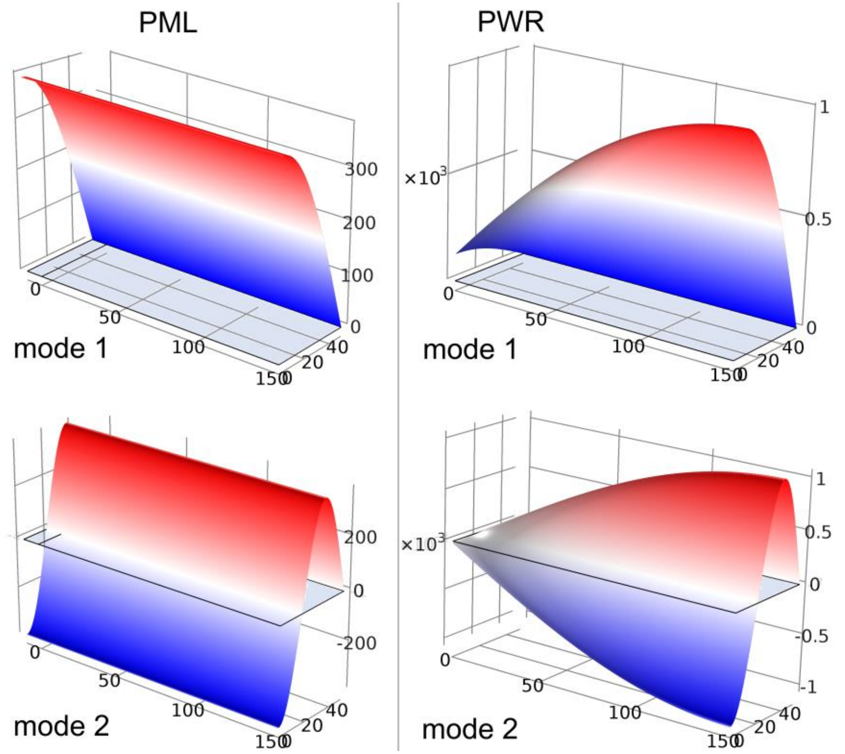
ξ = Dimensionless coordinate, [0,1]

s = Scaling parameter, p = curvature parameter

Λ = Typical wavelength parameter

Results: PML vs. PWR - Modal analysis

- Case (a): Rigid dam with perfect bottom reflection (α=1)



Plot: First two eigenmodes, height is proportional to water pressure

✗ **PWR:** Symmetry violated

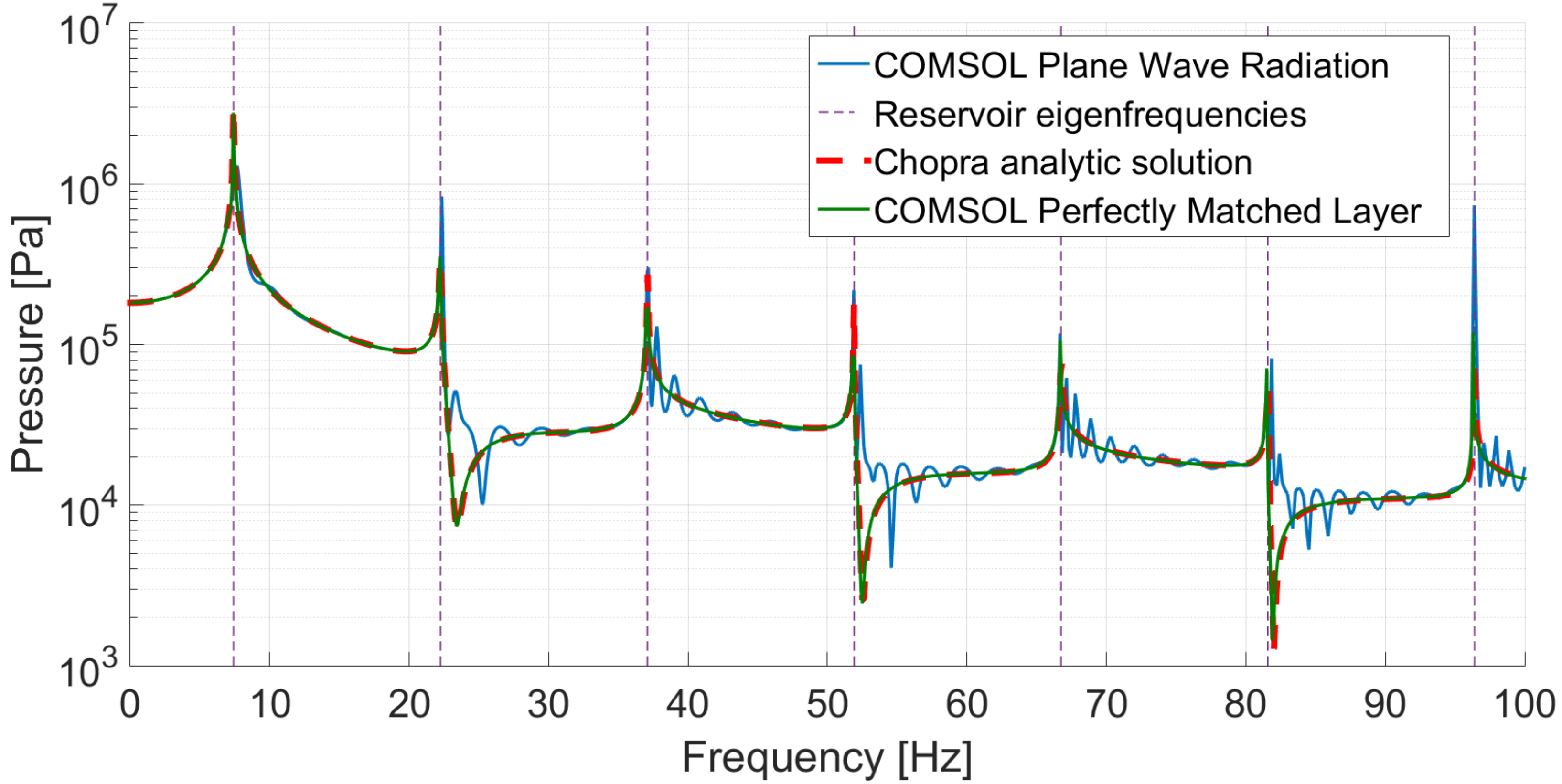
✓ **PML:** Symmetry respected

Reservoir depth = 50 m

Eigen mode	Analytic $(\frac{nc}{4h})$ [Hz]	PWR [Hz]	PML [Hz]
1	7.407	7.817	7.407
2	22.221	22.386	22.222
3	37.035	37.164	37.036
4	51.849	51.976	51.851
5	66.663	66.720	66.667

Results: PML vs. PWR – Frequency response

- Case (a): Rigid dam with perfect bottom reflection (α=1)



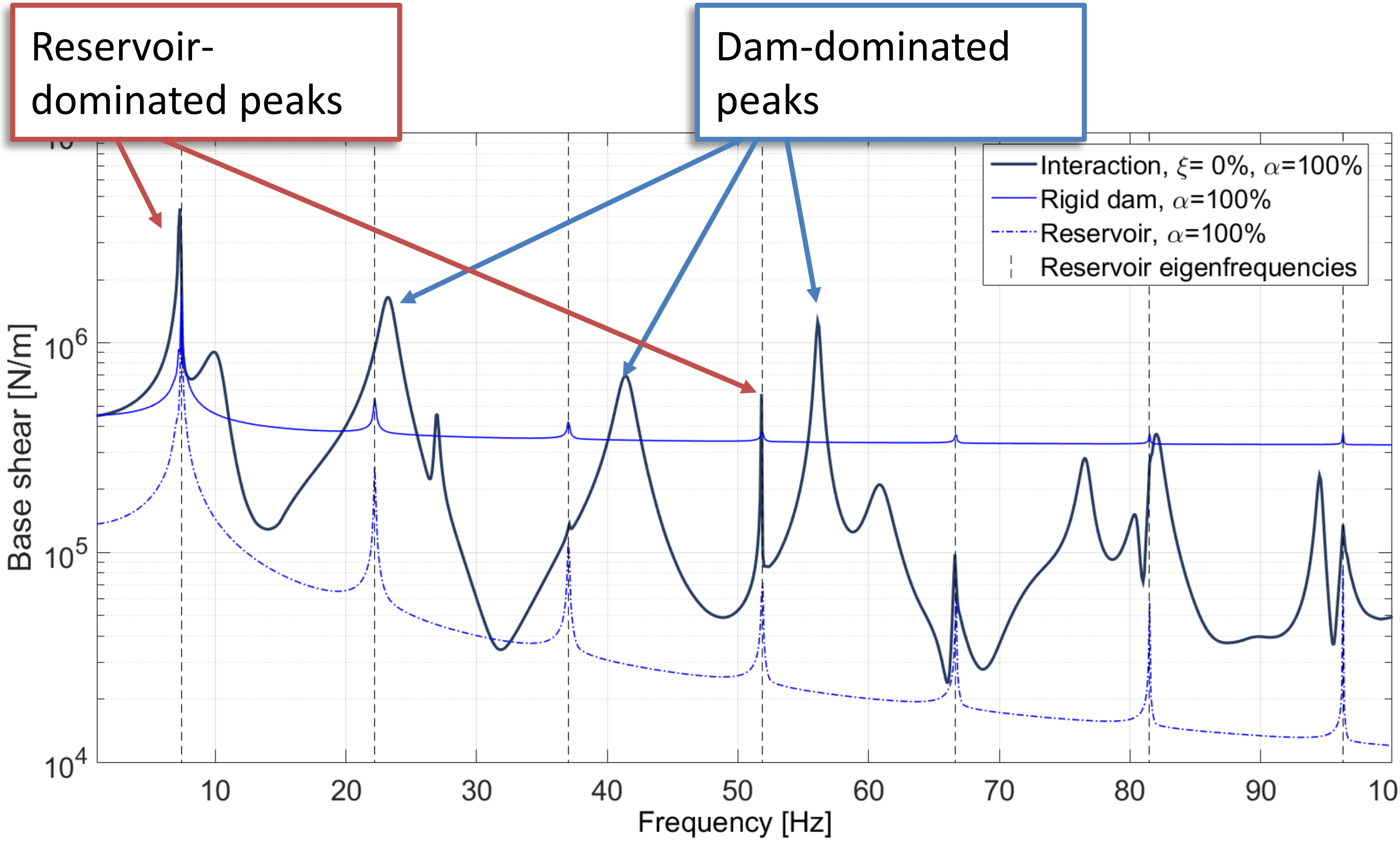
Plot: Total hydrodynamic pressure at the base of the dam.

✗ **PWR:**
Introduction of spurious peaks

✓ **PML:**
Correct modeling of the infinite-length reservoir

Results: Multphysics interaction

- Case (b): Deformable dam with perfect bottom reflection (α=1)



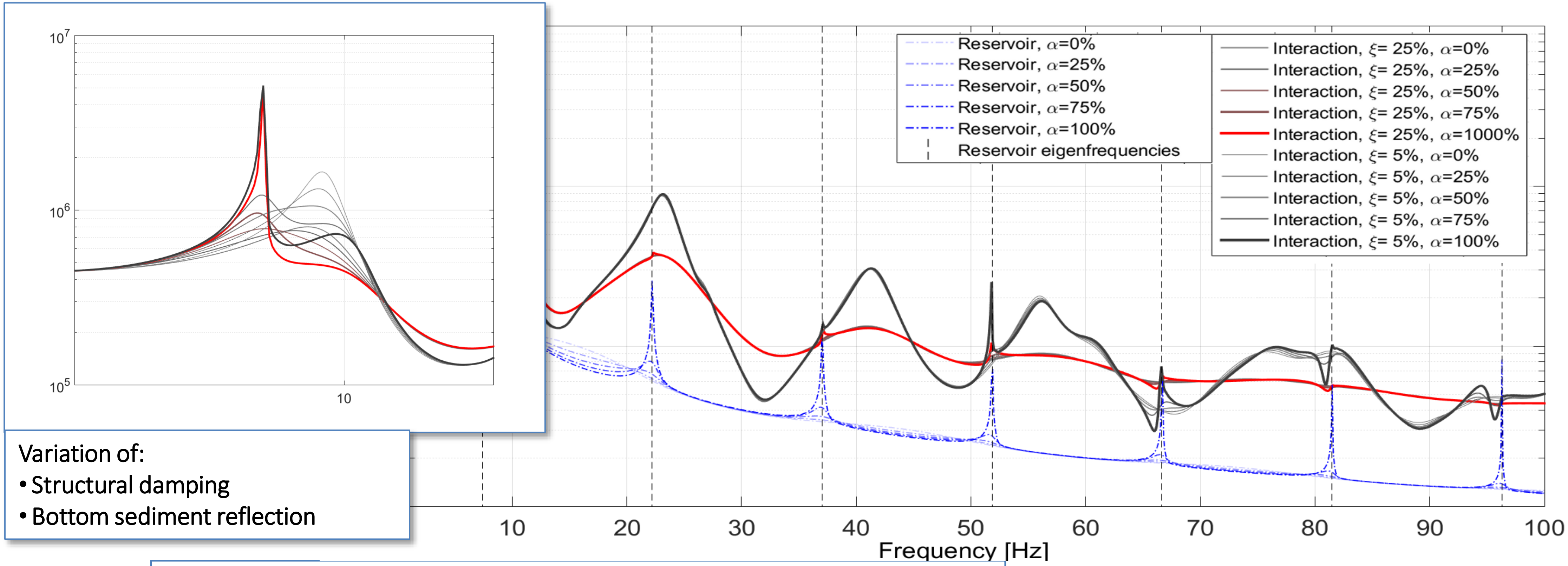
Reservoir-dominated peaks

Dam-dominated peaks

Plot: Horizontal base reaction at the dam foundation.

Results: Multphysics interaction

- Case (b): Deformable dam with various reflection coefficients (α)

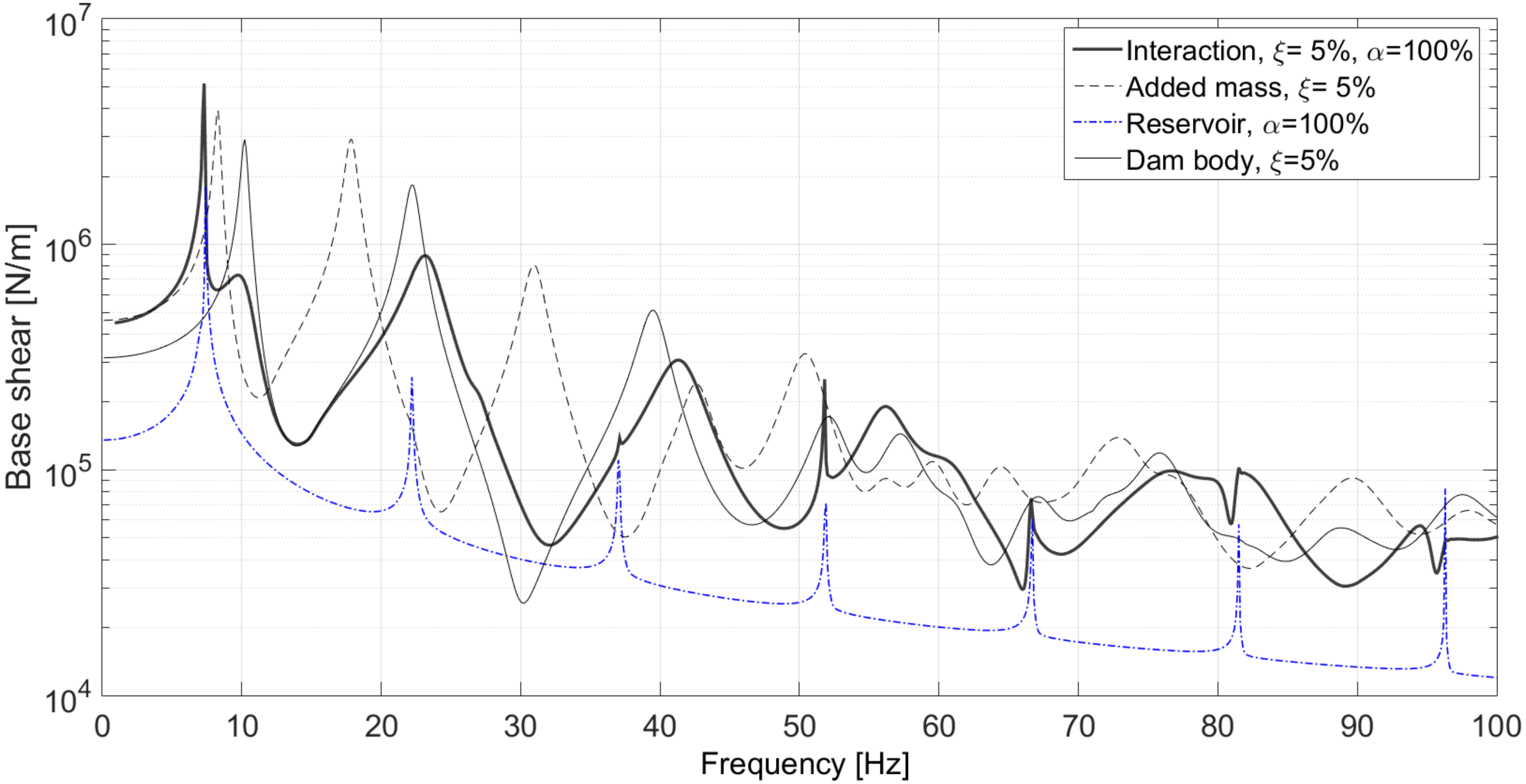


Variation of:
- Structural damping
- Bottom sediment reflection

Plot: Horizontal base reaction at the dam foundation.

Results: Multphysics interaction

- Case (b): Deformable dam compared to the added mass method



Plot: Horizontal base reaction at the dam foundation.

Added mass:

- ✗ Inaccurate Identification of other peaks
- ✓ Approximate Identification of the first mode
- ✓ Accurate value of base shear For low frequencies

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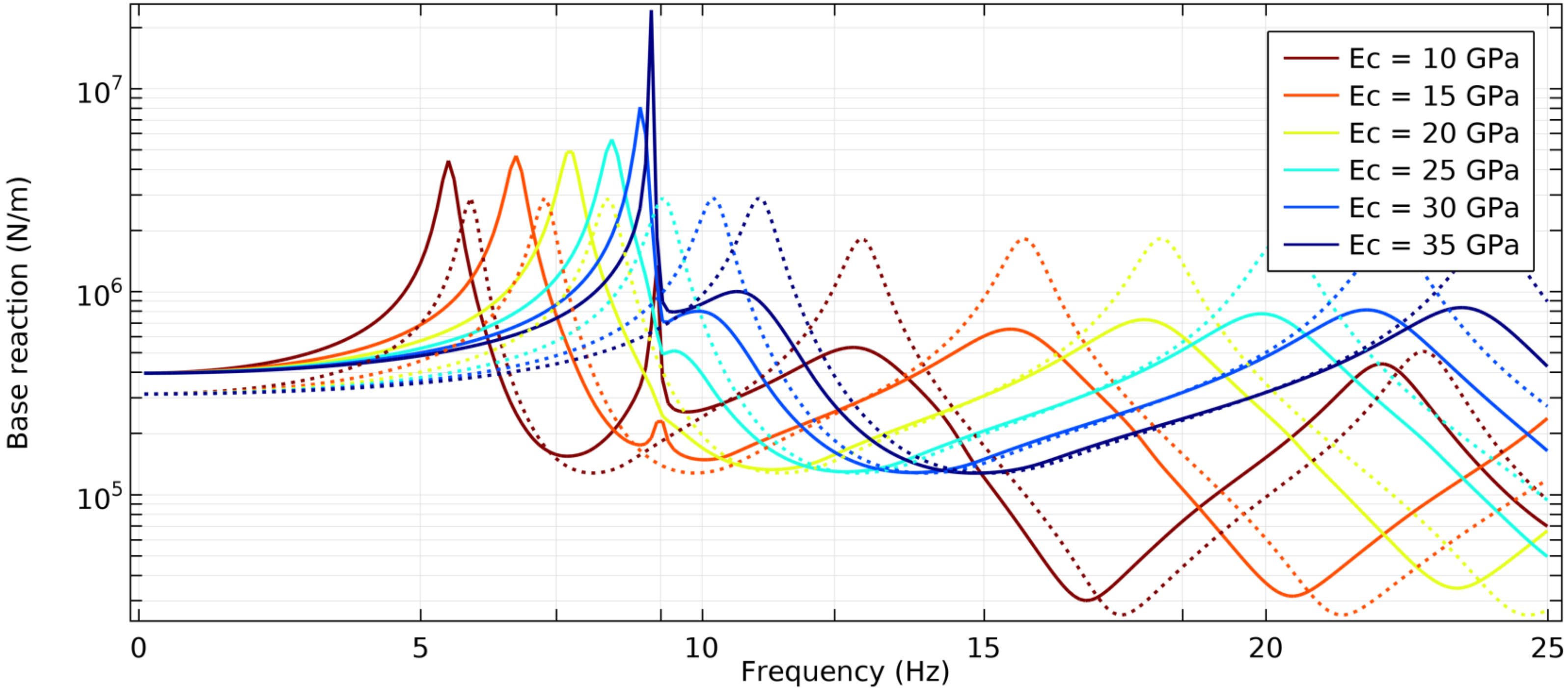
Results: Multphysics interaction

- Case (b): Variation of dam Young’s modulus



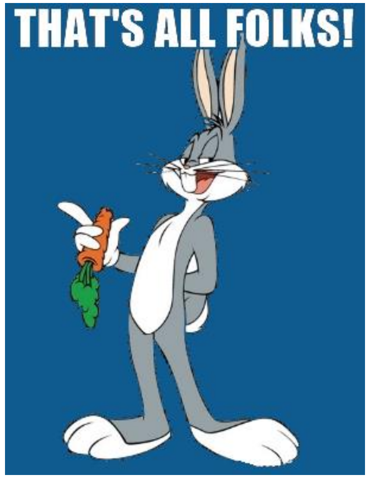
Plot: *Horizontal base reaction at the dam foundation.*

Added mass:

✗ Constant frequency shift of the peak

Acoustic-structure interaction:

✓ Accurate representation of the nonlinear Frequency shift



Thanks for your attention!