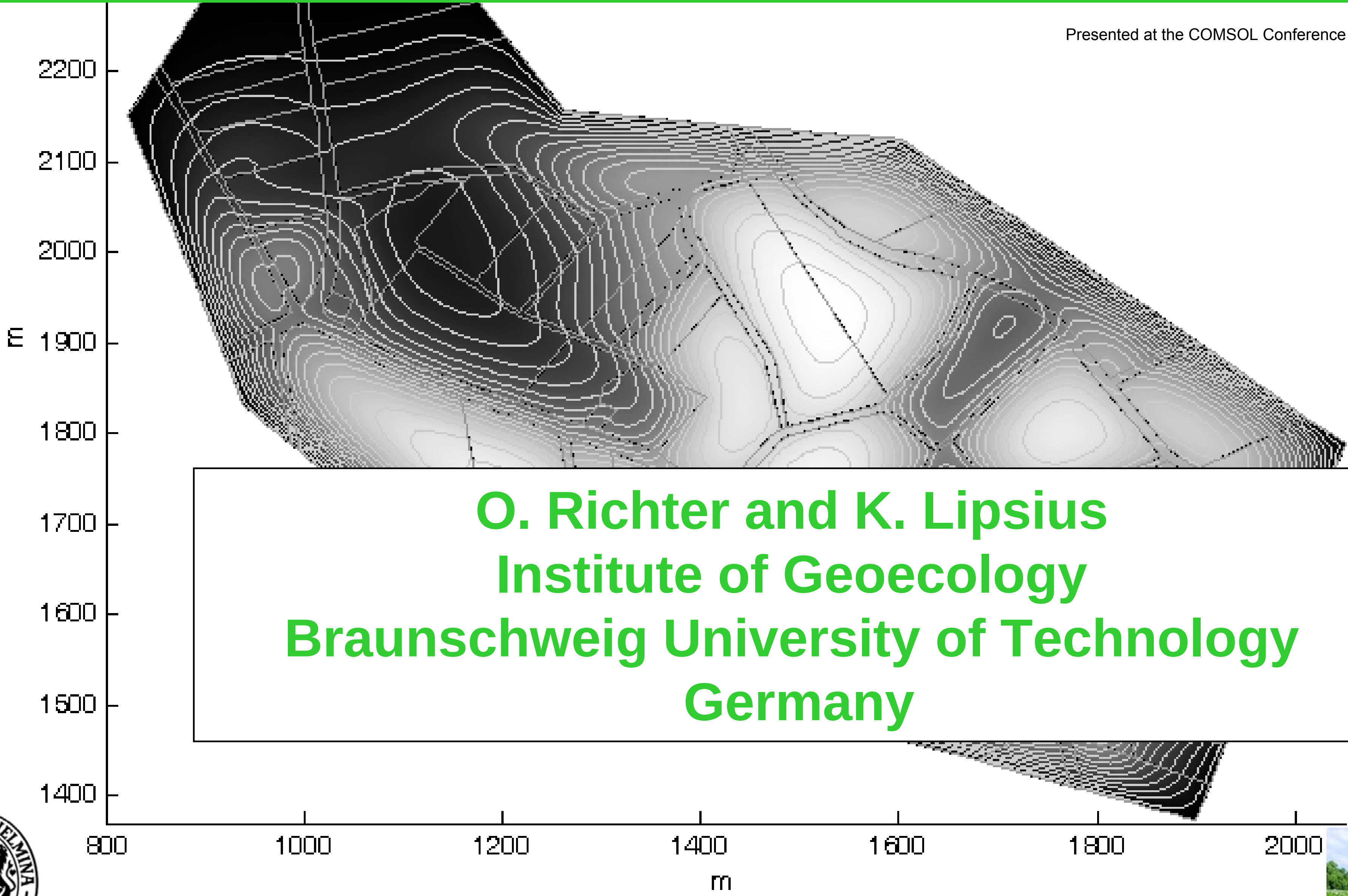
Modelling Dispersal of Genetic Information in Structured Agricultural Landscapes with Partial Differential Equations

Presented at the COMSOL Conference 2008 Hannover



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The biosafety problem: outcrossing from GMO crops

wind?

phenology?

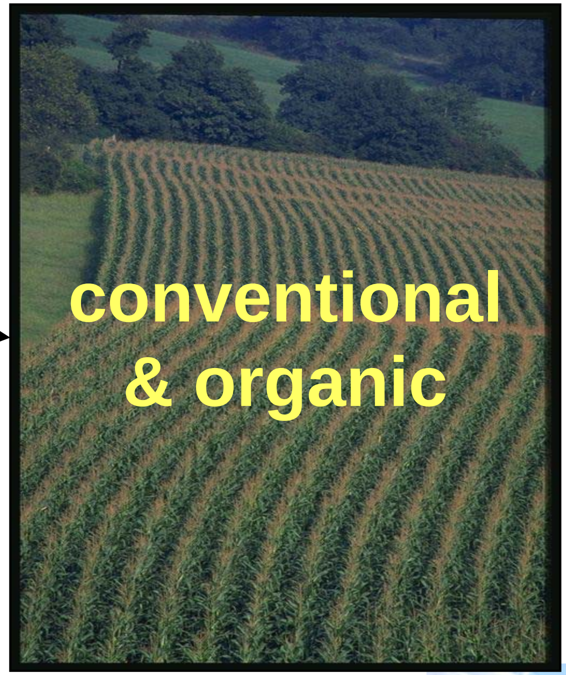
atmospheric stability?

model output/ threshold value?

flowering period?

long distance transport?

geometry?



land use?

short distance transport?

scale?

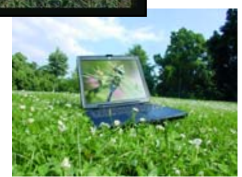
crop rotation?

obstacles?

weather?

landscape sructure?

soil?



Modelling gene flow in landscapes with partial differential equations

Advantages:

- The processes can be cast into a highly aggregated mathematical form
- Number of parameters is limited
- Geometries of heterogeneous landscapes can be linked to finite element nets

General approach

$$\frac{\partial N_i}{\partial t} = L[N_i] + f_i(N_1, N_2 \ldots N_n) \qquad i = 1 \cdots n$$

space operator

reaction terms: population dynamics, genetics

$$L[N] = \nabla D(N) \nabla N - \nabla \vec{v} N$$

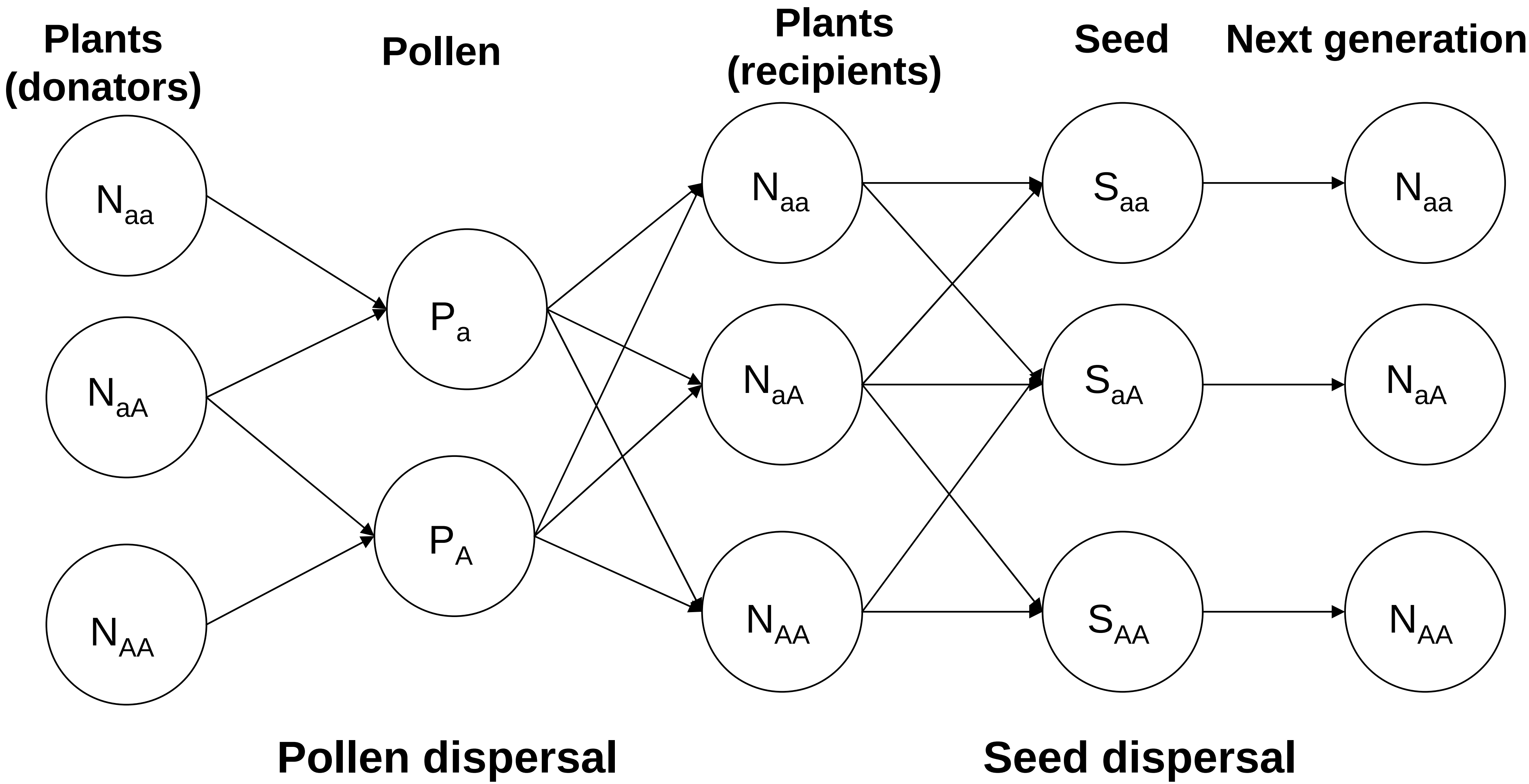
Diffusion

Convection

$$\frac{\partial N}{\partial t} = \nabla \cdot D(x,y) \nabla N + rN(1 - \frac{N}{K(x,y)})$$

Simple example: linear diffusion and logistic growth

Conceptual model



Equations

Pollen $$\frac{\partial P_i}{\partial t} = \nabla \bullet \left(D_{P_i}(x, y) \nabla P_i - \vec{u} P_i \right) + f_i(N_{ij}, t)$$

Seed $$\frac{\partial S_{ij}}{\partial t} = \nabla \bullet \left(D_{S_{ij}}(x, y) \nabla S_{ij} - \vec{v} S_{ij} \right) + f_{ij}(N_{ij}, P_i, t)$$

Plants $$\frac{\partial N_{ij}}{\partial t} = r \cdot N_{ij}\left(1 - \frac{N_{ij}}{K(x, y)}\right) + \alpha_{ij} \frac{S_{ij}}{S_{ij} + K_{ms}}$$

i=a, A
j=a, A

Interaction terms

Pollen

$$f_a = \beta_p\left(\frac{N_{aA}}{2} + N_{aa}\right) - \mu_p \cdot P_a$$

$$f_A = \beta_p\left(\frac{N_{aA}}{2} + N_{AA}\right) - \mu_p \cdot P_A$$

$$f_{aA} = \beta_s\left(\frac{N_{aA}}{2} + N_{aa}\right)\left(\frac{P_A}{P_a + P_A}\right) + \beta_s\left(\frac{N_{aA}}{2} + N_{AA}\right)\left(\frac{P_a}{P_a + P_A}\right) - \mu_s \cdot S_{aA}$$

Seed

$$f_{AA} = \beta_s\left(\frac{N_{aA}}{2} + N_{AA}\right)\left(\frac{P_A}{P_a + P_A}\right) - \mu_s \cdot S_{aa}$$

$$f_{aa} = \beta_s\left(\frac{N_{aA}}{2} + N_{aa}\right)\left(\frac{P_a}{P_a + P_A}\right) - \mu_s \cdot S_{aa}$$

Imposing spatial and temporal structures

- The system consists of PDE‘s and ODE‘S
- ODE‘s model plant growth
- PDE‘s model dispersal of pollen and seed
- The capacity terms $K(x,y,t)$ and the diffusion coefficients $D_{ij}(x,y)$ are linked to spatial structure
- Time dependent coefficients e.g. $ß_s(t)$,$r(t)$ define temporal patterns

Processes implemented

- Crop rotation: capacities on fields change in time
- Crop management: sowing, harvest, and other management practices influencing e.g. seed production, seed survival
- Transport: Diffusion and convection as Resistance against transport

Parameter derivation

- Parameter set for each genetic variety
- Capacity, seed production, pollen production, seed survival, diffusion, convection
- Describes environmental fitness over time
- Depends on crop rotation and crop management practices

Linking geoinformation and model

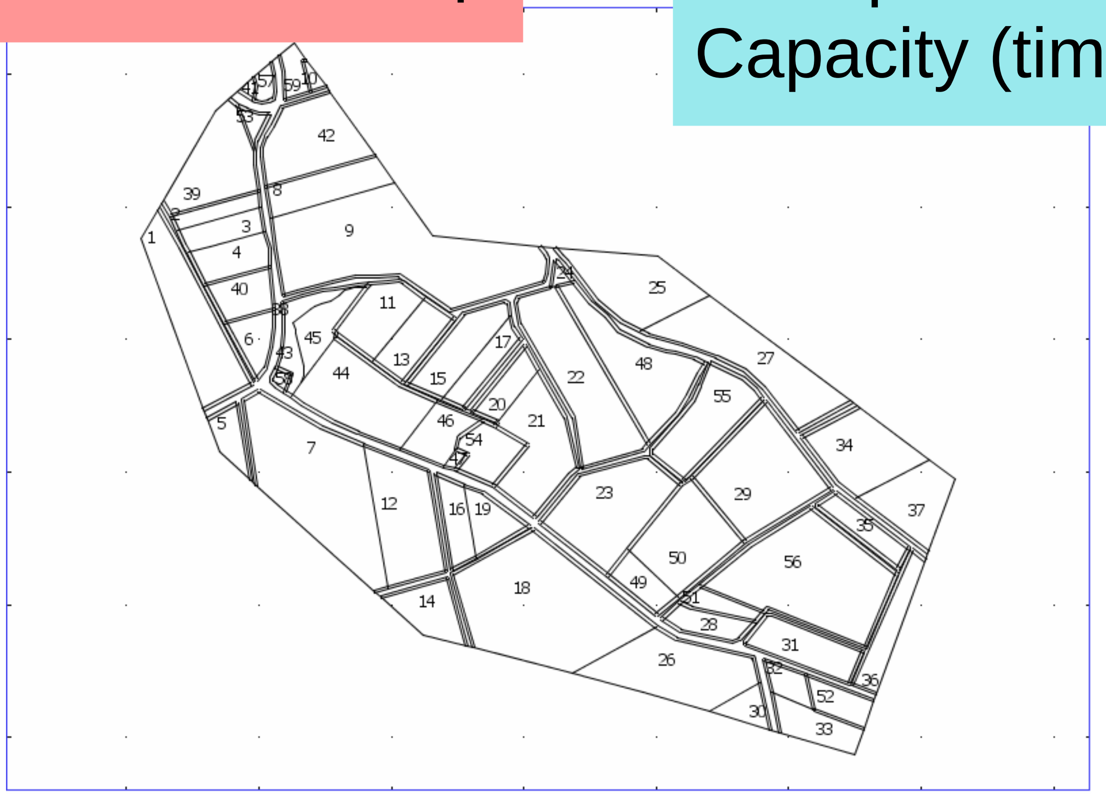
$$\frac{\partial N}{\partial t} = \nabla \cdot D(x,y)\nabla N + r(x,y,t)N\left(1 - \frac{N}{K(x,y,t)}\right)$$

⇧

Resistance map

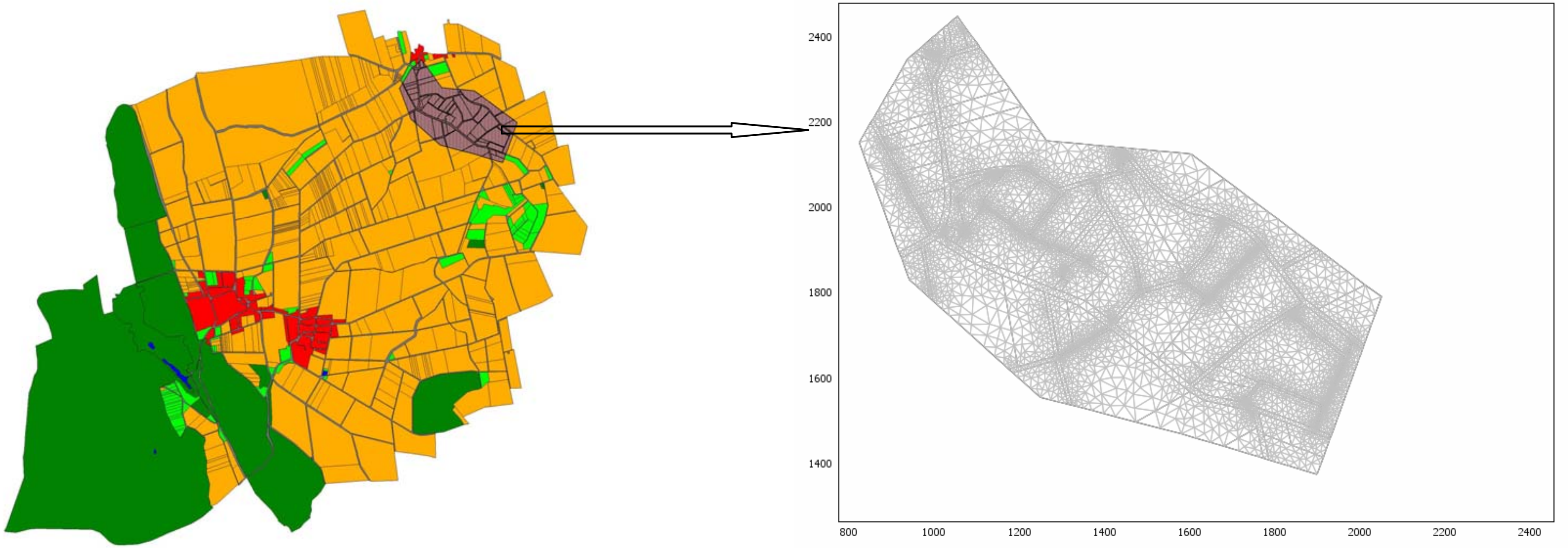
⇧

Map of environmental Capacity (time dependent)



Import of a Landscape to Comsol

ArcToolbox: „Export to CAD“ tool

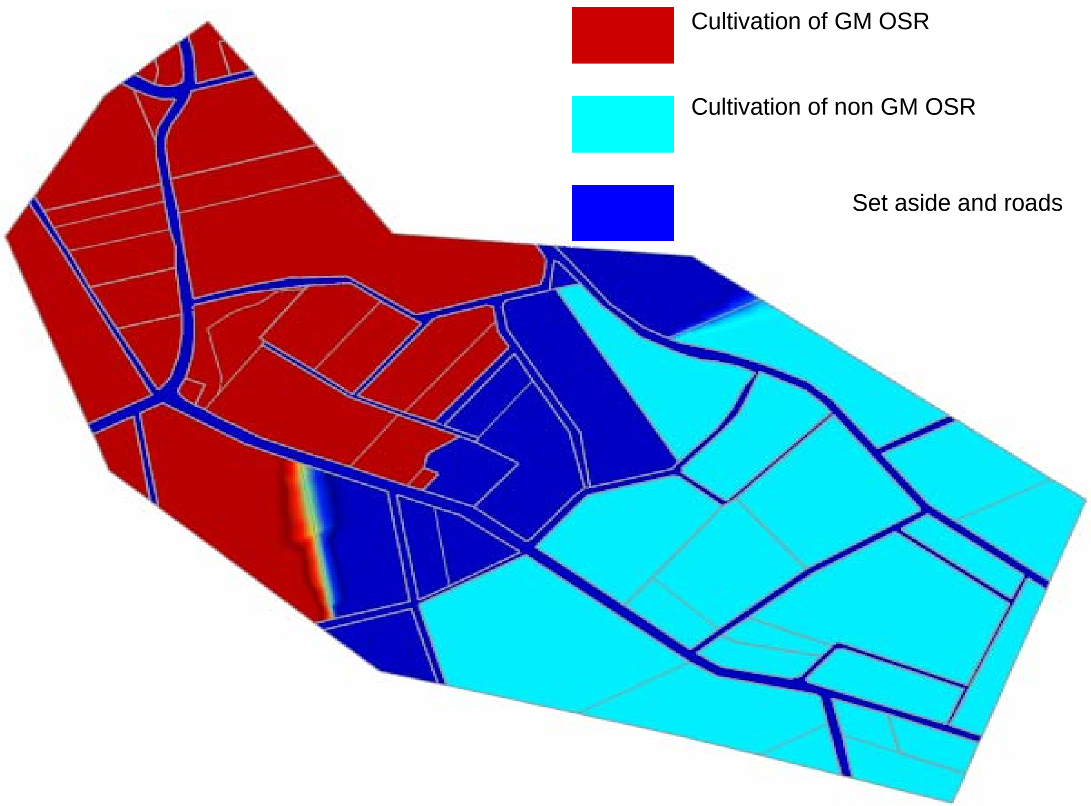


Finite element mesh

Theoretical biosafety study of a genetically modified crop

- The crop under study such as oil seed rape has been made tolerant to non-selective systemic herbicides
- At the phenotype level, resistance affects mortality rates under herbicide applications.
- **Question: how does the management of roadsides influence the propagation of unwanted genetic information?**

Scenario: Initial allocation of land use and crops



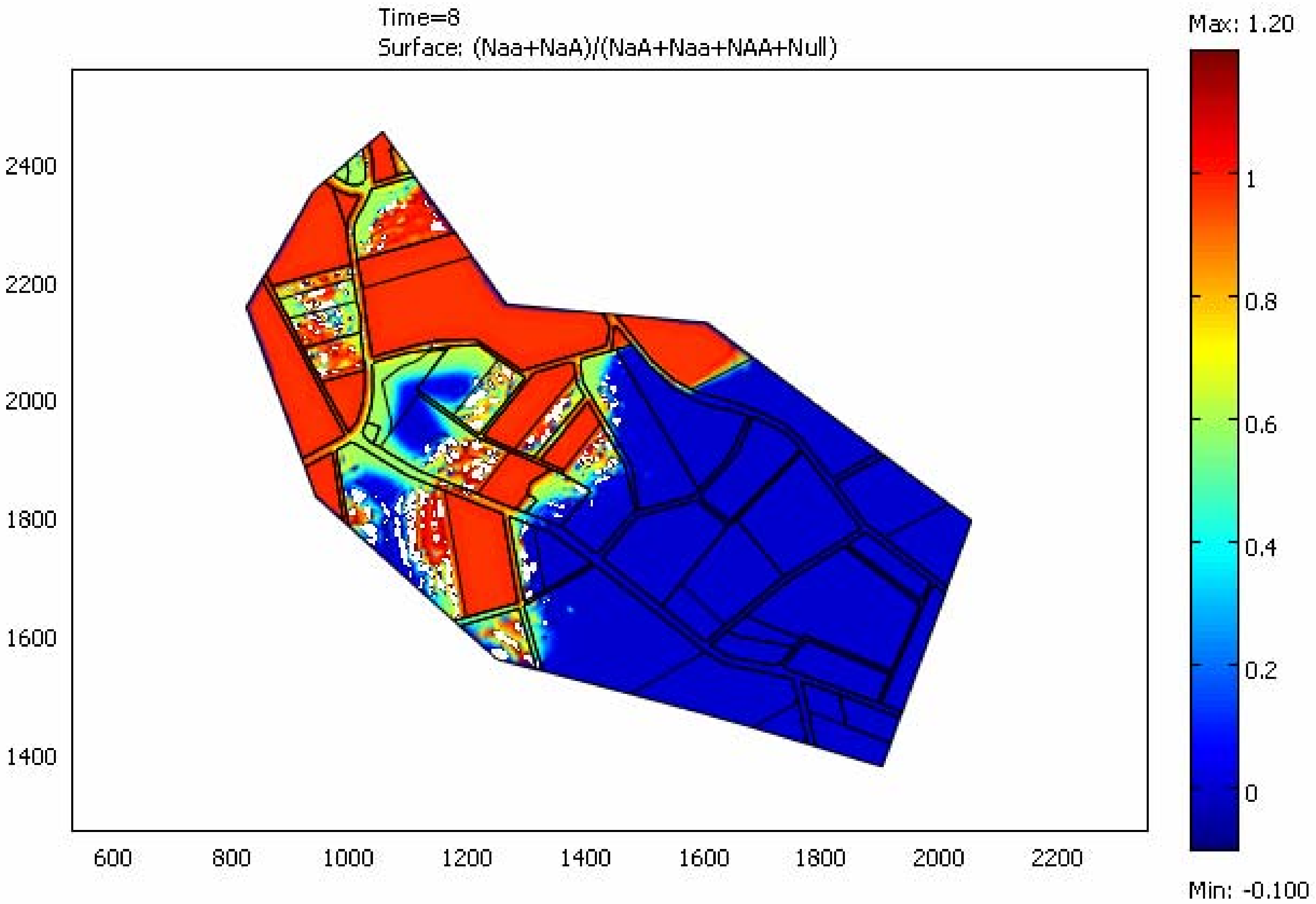
Scenario parameters

- **Crop rotation**: 3 years: osr, ww, wb

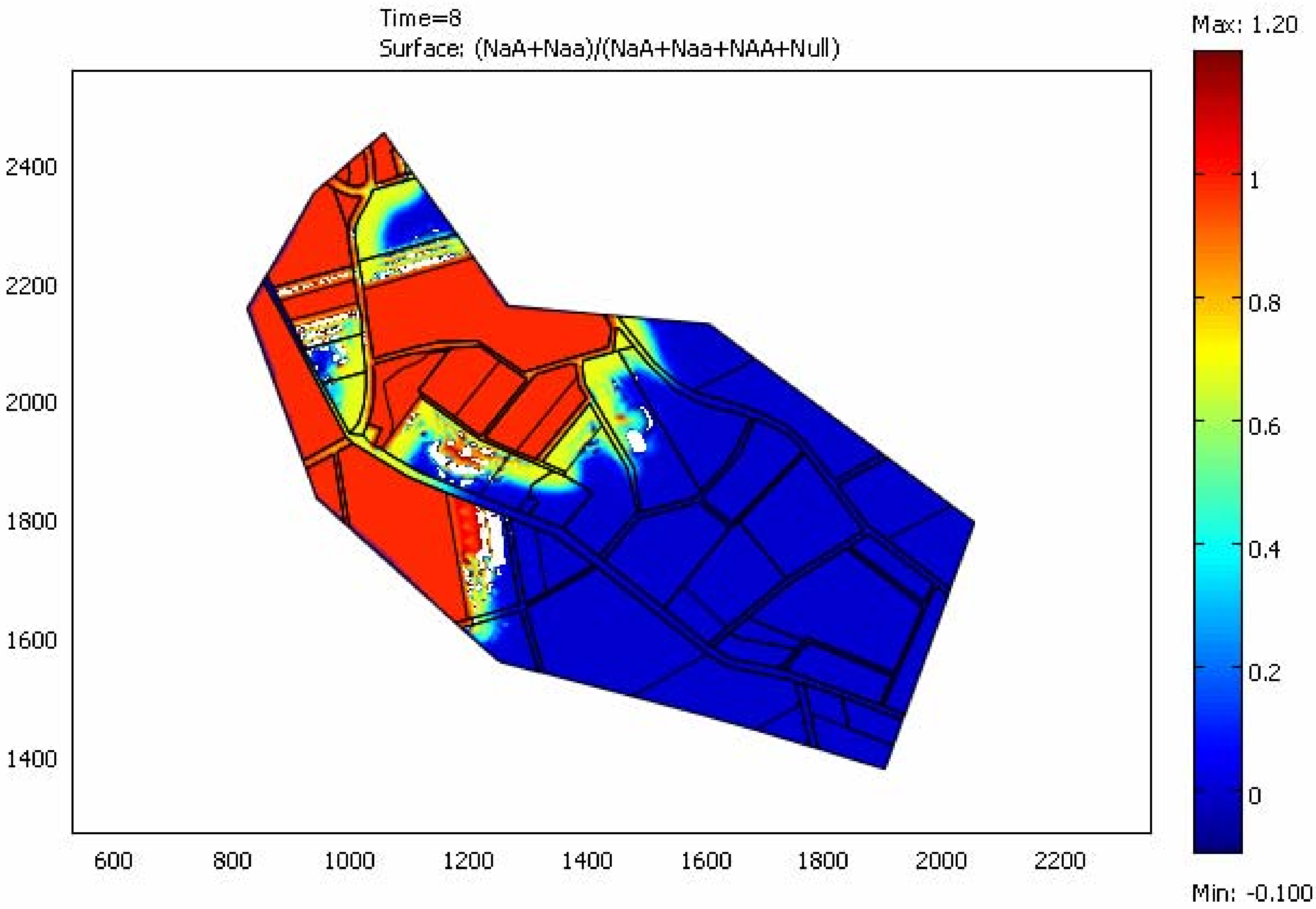
- **Allocation of parameters**

- **Capacity:** woodland: low, fields: time dependent, lanes: high/low
    - **Spatial resistance:** woodland: high, fields: middle, lanes: low

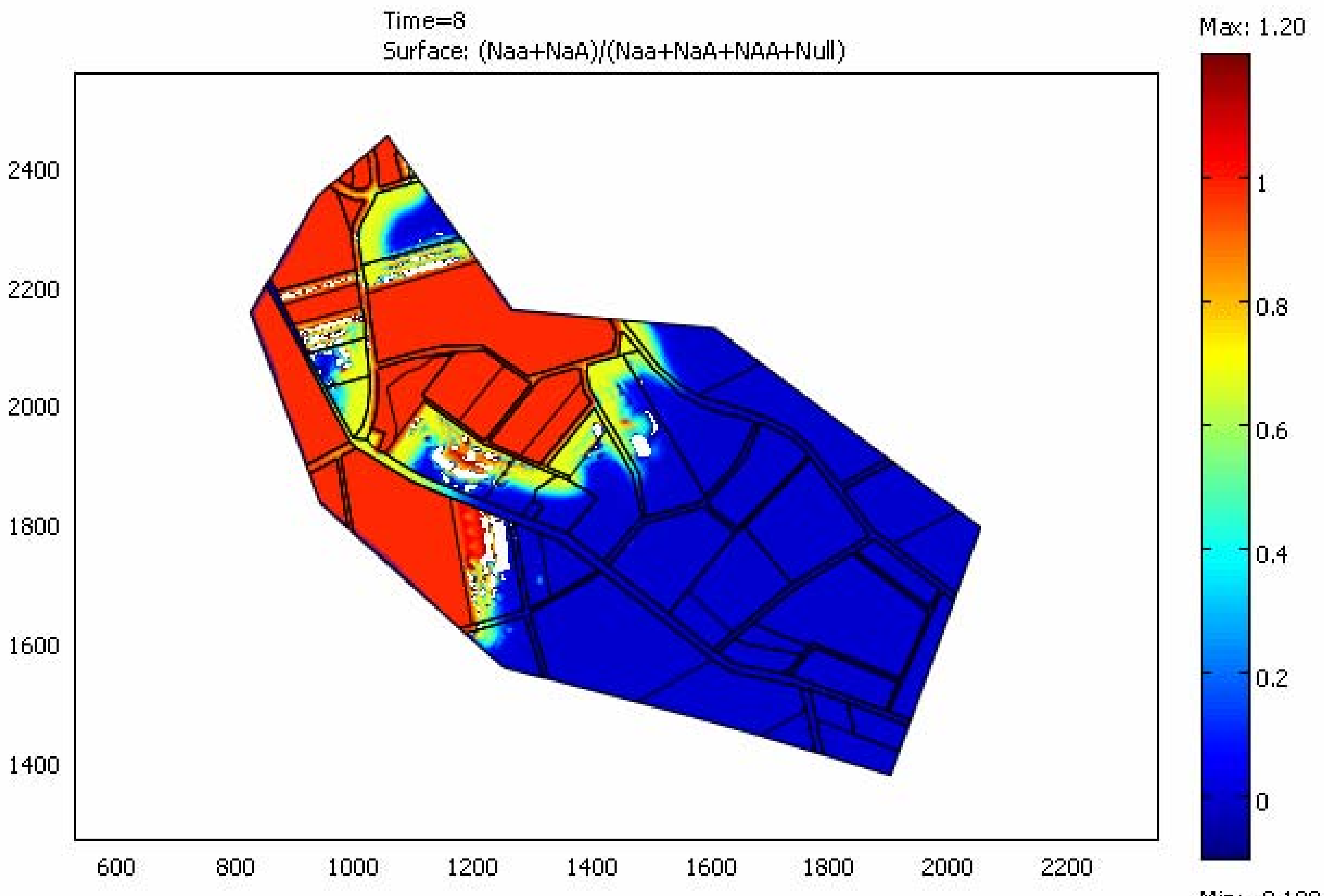
GM ratio with no herbicides in roads with no isolation distance!



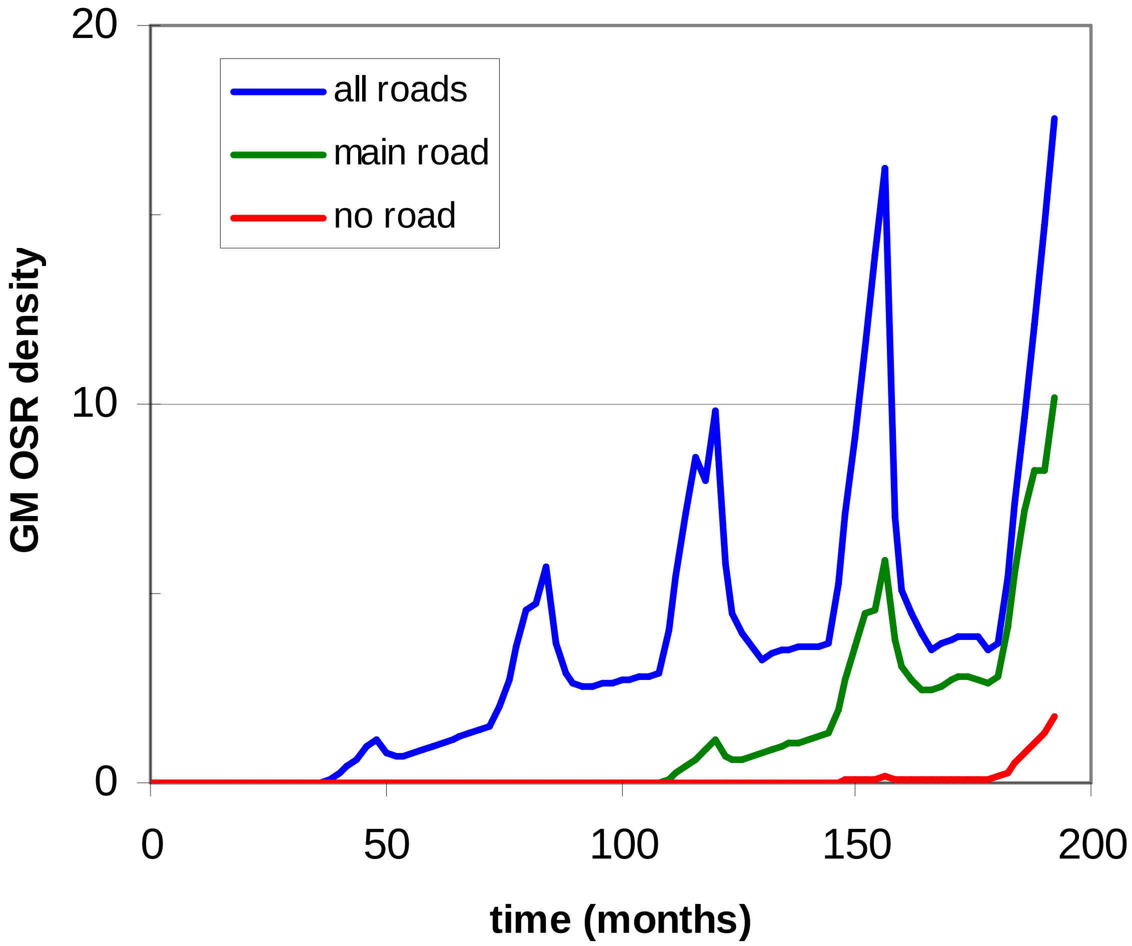
GM ratio with herbicides along main road (e.g. railway track) with isolation distance



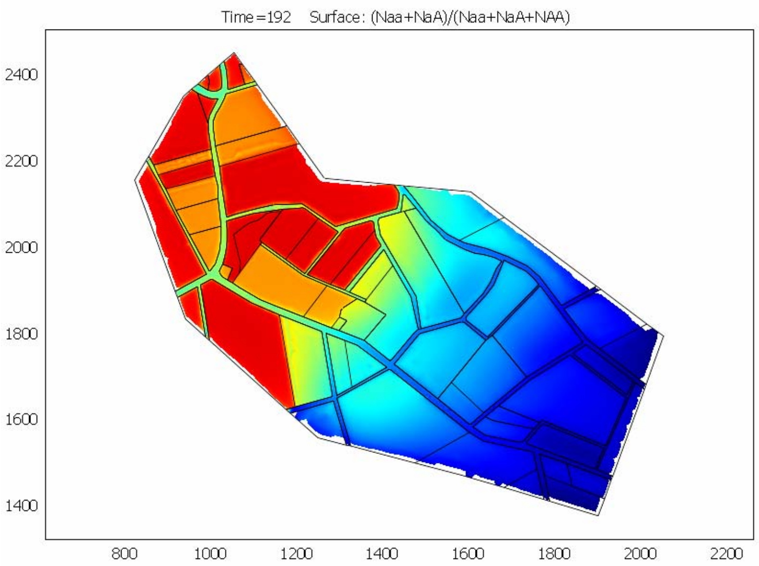
GM ratio with herbicides along all roads with isolation distance

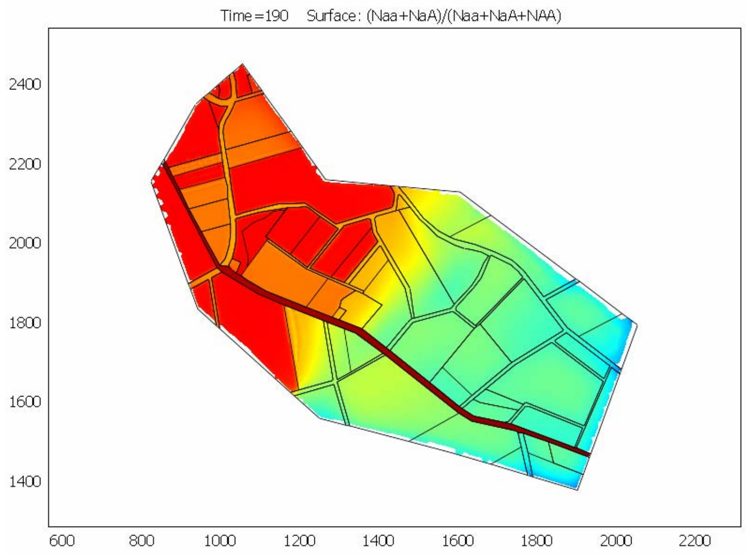


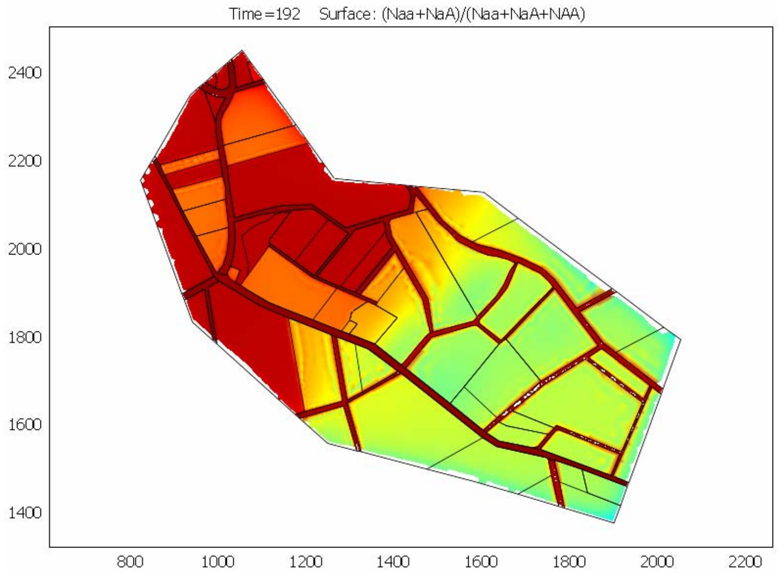
GM densities in a non GM field for three scenarios of herbicide applications along roads



Herbicide application along roads facilitates spread of GM species







Summary and Outlook

- The coupling of PDEs and population dynamics with geoinformation provides a model framework for gene flow at landscape scale.
- It is easliy implemented into COMSOL multiphysics
- The highly aggregated approach results in only few model parameters, so parameter estimation from field observations is feasible. This is the next step to be taken to endow the model with realistic parameter sets.
- Possible applications are optimization of crop rotation and crop allocation schemes and evaluation of isolation distances.

Thank you for your attention

Selected parameter values

Parameter	GM OSR O_{AA}	Conv. OSR O_{aA}
β_S	0.08	0.11
D_S road	500	500
D_S field	25	25
K road	2	2
K conv field	120	120
K GM field	120	2
K WW	5	5
K WB	4	4