Natural Convection Effects On The Solidification In Cylinders At Different Filling Percentages

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Introduction

The problem of liquid solidification in a partially-filled circular cylinder is encountered in a number of industrial processes such as the oil and gas industry, food industry, thermal energy storage systems, casting of metals and alloys, and many others. The efficacy and performance of many of these systems often depend on the specific behavior of the fluid as it freezes. In enclosures, specifically, it also depends on the level of filling. A deeper understanding of the progression of the phase change process is crucial in understanding how the various geometric and fluid parameters and boundary conditions affect the process.

Ever since the pioneering work of Stefan [1], the physics and process of solidification in fluids have been the target of extensive study, theoretical, numerical and experimental. The “basic solution” presumes that energy transfer is due to conduction only [2]. When density variations are present, coupled with a gravitational field, natural convection currents cause the process of solidification to deviate from the conduction-dominated one. This difference in behavior could be critical for a number of these applications, which warrants further scrutiny.

For one, the interface between the solid and liquid phases need to be tracked accurately. This is due to the difference heat capacity properties of the two phases, in addition to the buoyancy effect present in the liquid phase.

Mahdavi et al. [4] studied entropy generation in heat transfer when an inner cylinder of porous material is used, in contrast to a constant-heat flux inner cylinder. The effect of the position of the inner cylinder, among other things, was found to affect the thermal performance of the cylinder, which validates the importance of studying the effect of the filling percentage. Alawadhi and Bourisli [5] investigated the solidification of water in an annulus and the effect of peak density on flow structure. High resolution finite element solutions showing the details of hydrodynamic and thermal fields demonstrated the sensitivity of the solidification process to geometry. Ismail et al. [6] presented results for different Dean numbers to show the effect of pipe curvature on solidification. Alawadhi [7] also investigated the solidification of water inside elliptical enclosures at different aspect ratios and concluded that the solidification times decreased considerably with increasing aspect ratio. On the other hand, he observed that the inclination of the enclosures had negligible effect on the solidification time.

In the current paper, the effect of different filling percentage of the cylinder is investigated for a number of Rayleigh numbers. Varying Rayleigh number can be shown to measure, for example, the effect of using different fluids, different geometry, different temperature difference, or a combination thereof. The progress from fully-liquid to fully-solid is shown with time and a number of velocity and phase contours are used to elucidate the difference the filling percentage makes. The time to full solidification is also noted and related to the nature of the progress.

Problem Definition and Governing Equations

The solidification problem inside a two-dimensional cylinder of diameter D with different filling levels is investigated numerically using the finite element method. The different filling levels span the range from 20% (*i.e.*, 80% air above the liquid) to 100% (fully liquid).

The solidification phenomenon in general is governed by the conservation of mass, momentum and energy equations for an incompressible, Newtonian fluid,

$$\nabla \cdot \mathbf{u} = 0$$

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u}\right) = -\nabla p + \mu \nabla \cdot \left(\nabla \mathbf{u} + (\nabla \mathbf{u})^T\right) + \rho \mathbf{g}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T\right) = \frac{\partial p}{\partial t} + \mathbf{u} \cdot \nabla p + k \nabla^2 T$$

where it is understood that these equations apply to each phase according to the mass fraction in the fluid element. In addition, a phase-change equation is solved for the fluid/solid phase change.

Hydrodynamically, the no-slip boundary condition is enforced at the walls of the cylinder. The zero-shear stress at the air/water interface is also enforced, whenever applicable. Initially, the velocity is zero everywhere. Thermally, the (zero-thickness) wall experiences a sudden drop in temperature at (time) $t = 0$, from the initial temperature of $T_{initial}$ to a constant temperature T_{cold} that is lower than T_m, the phase change temperature of the fluid.

The Rayleigh numbers examined were all below 10^9, so the flow is assumed to be laminar. Rayleigh number is defined as,

$$Ra_D = \frac{g\beta\Delta T D^3}{\nu\alpha}$$

where g is the gravitational constant, β is the coefficient of thermal expansion, α is the thermal diffusivity, and ν is the kinematic viscosity. The temperature difference, $\Delta T = T_{initial} - T_{cold}$, is taken to be the difference between the cylinder wall temperature and the initial water temperature. At each filling case, several temperature difference values between the fluid initially and the solid, cold wall are investigated. This is the way Rayleigh number was changed, accounting for the intensity of the natural convection present.

Numerical Model and Simulation Steps

The governing equations are discretized using the finite element method by way of COMSOL Mutiphysics. A number of modules were used in solving these equations—specifically, the *single-phase laminar flow (spf)* and the *heat transfer in fluids (htf)* modules. Within each module, a few fundamental components are necessary to effect the desired domain, boundary and initial conditions. The effect of natural convection is captured by including the effect of *gravity* in the laminar flow physics. This, combined with the varying material density (as discussed below) enables the model to differentiate between the solid and liquid phases in terms of their response to gravitational acceleration—thus giving rise to buoyancy and natural convection currents. In addition, a *phase-change* node is added under the heat transfer physics to the water domain where the viscosity varies between that of water and “infinity” depending on the temperature. The solidification effect is captured by defining two materials, a liquid and a solid, for the domain in the material node, and having the viscosity of the “solid” phase be 10^{22} times that of the fluid. Properties within each material is assumed to be constant.

The interval over which phase change occurs was set to 0.5˚C. The surface tension effect in the air-water interface was not modeled because of the insignificant effect it has compared with the natural convection currents in the water. Finally, due to the small velocities involved, no viscous dissipation was considered. The COMSOL built-in properties and constants where used throughout.

The built-in mesh generator was used to furnish the spatial discretization of the flow domain. To establish grid-independence, a few meshes were used on the basic, 100%-filling-level case and both the time to complete solidification and the rate of heat transfer through the walls were noted. It was found that, finally, an extremely fine mesh yielded changes in these two quantities of less than 1% compared to the very fine mesh, but the extremely fine mesh was used anyway. That mesh contained 21,132 domain elements, 655 boundary elements, and 49,240 degrees of freedom. The default values for time discretization, Newton iterations, residuals, and all other solver options were used.

A single, elaborate COMSOL parametric study was set up to compute all the results needed in this study, with a few options deactivated during the grid independence tests.

Results and Discussion

Numerical results show an interesting dependence of the flow and phase fields on Rayleigh number. For example, left sides of Figures 1(a) through 1(d) illustrate how the water solidification process depends on the intensity of natural convection currents, represented by Rayleigh number, for the 25% filling case. The higher the Ra, the more vigorous the currents in the water—left side of Figure 1(d). The enhanced circulation at higher Ra exposes warm water from the center of the liquid section to the cold upper (free) surface. An upper thin layer of ice develops, which is not present at lower Ra values. The same effect, although more pronounced, can be seen when the filling percentage is 75%—left side of Figure 2(a) through 2(d). The gradients of the temperature for all these cases correspond well to the expected relative rates of heat transfer—right sides of the figures.

The resulting effect on the rate of solidification is also evident when looking at ‘time to complete solidification.’ The log-log Figures 3 and 4 show the solidification level reached as a function of time for a number of Ra values for the 25% and the 75% filling cases, respectively.

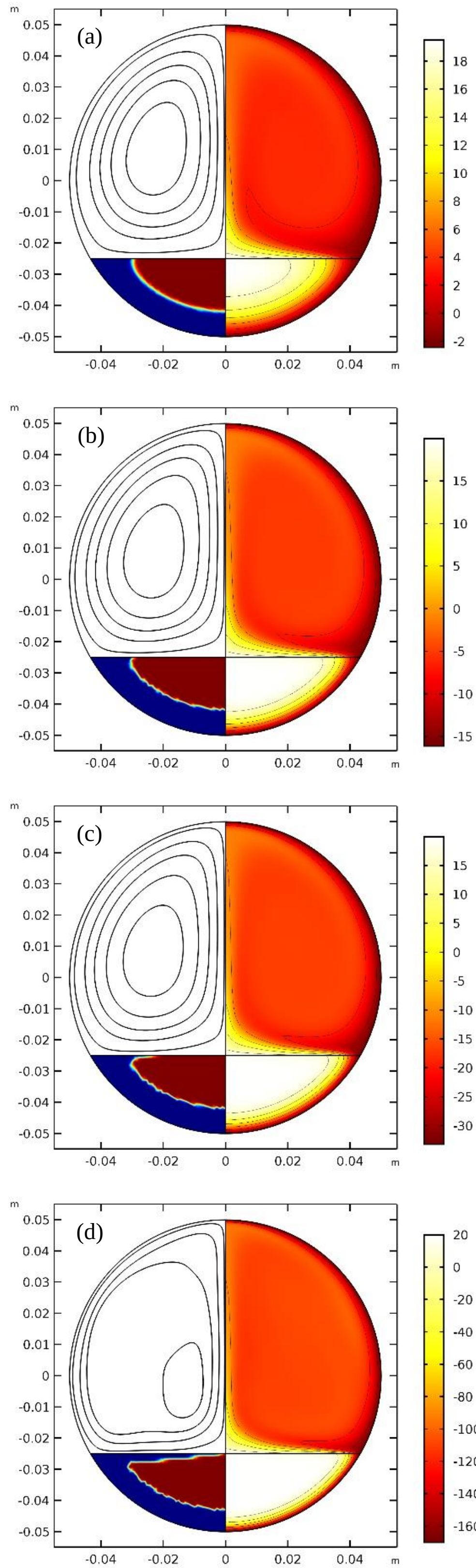

Figure 1. Phase (left) and temperature (right) contours for Ra = 10^7, $5x10^7$, 10^8, and $5x10^8$, for the 25% filling case.

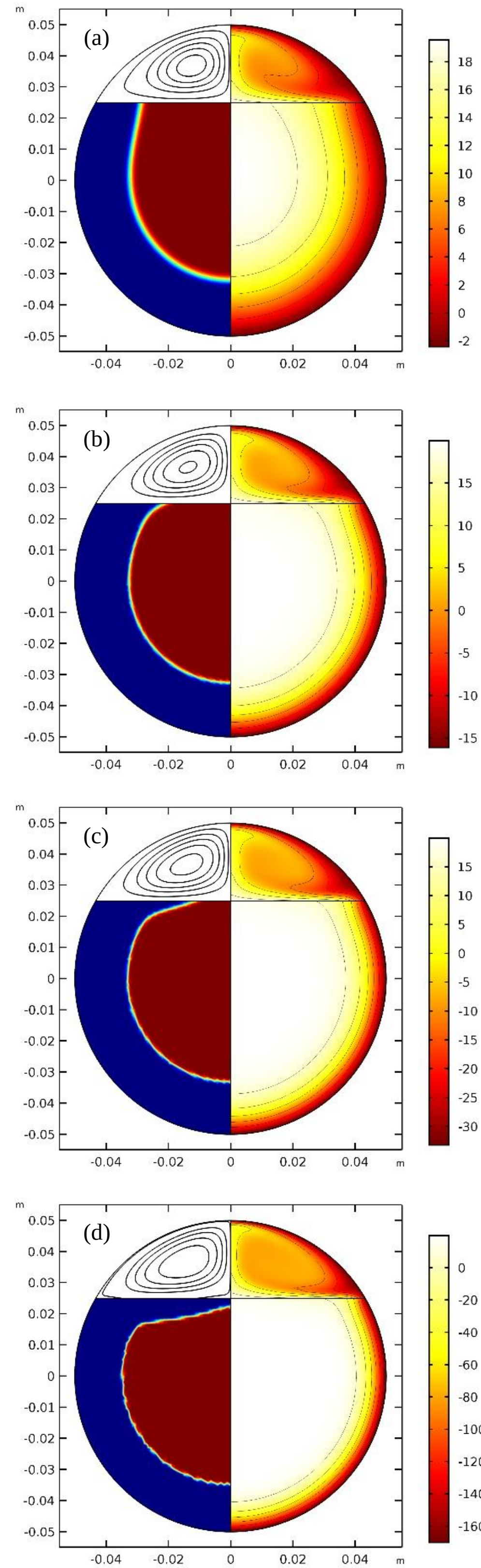

Figure 2. Phase (left) and temperature (right) contours for Ra = 10^7, $5x10^7$, 10^8, and $5x10^8$, for the 75% filling case.

One notices the banding together of the lines in Figure 4 at higher Ra, compared to those in Figure 3. This can be due to the added volume of liquid, and thus added shielding of the liquid layers at the center of the liquid region, for the higher filling percentage. But the primary reasons for this, we argue, is the smaller air volume above the free surface and the lost potential for natural convection currents to contribute to the solidification of the upper region. This confirms what was observed from Figures 1 and 2.

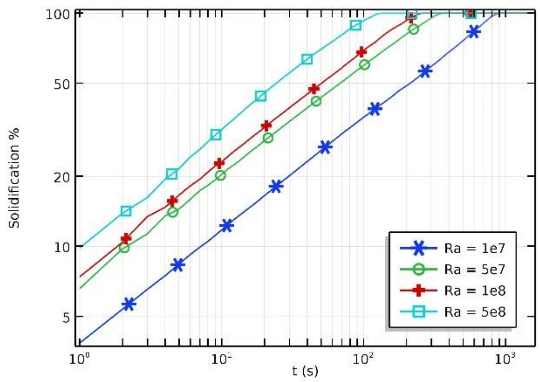

Figure 3. Solidification percentage vs. time, at different values of Rayleigh number for the 25% filling case.

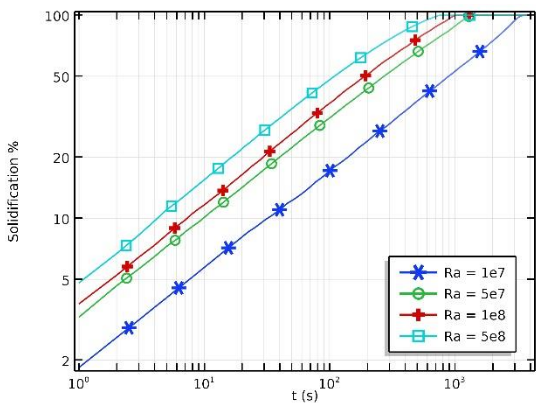

Figure 4. Solidification percentage vs. time, at different values of Rayleigh number for the 75% filling case.

As for the effect of the filling percentage on the solidification time, Figure 5 shows the log-log relation of solidification percentage versus time, for a few filling percentages. It can be observed that lower filling percentages resulted in progressively faster solidification. Part of this acceleration of solidification is naturally due to the smaller volume of liquid, but part of this apparently nonlinear relation is also due to the effect of the surface area afforded to conduction from the surface. It is quite possible that a hydraulic dimeter, defined using the wetted perimeter of the cylinder, could be used in Rayleigh number to eliminate this nonlinearity. This is the subject of ongoing research.

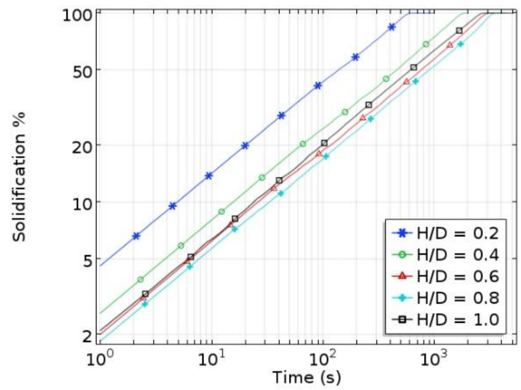

Figure 5. Solidification percentage vs. time for different filling percentages at Ra = 10^7.

Conclusions

In this work, the solidification process of a 2-D, partially-filled cylinder is investigated using COMSOL. The nature and progress of the water freezing is shown for several filling percentages and Rayleigh numbers. It was found that the time to full solidification depends greatly on the filling percentage and that this dependence is nonlinear. Moreover, the effect of Rayleigh number is clear in accelerating the process due to advection of warm fluid from the core to the peripheries. In terms of absolute numbers, we believe that the inclusion of the free surface motion will result in more accurate results for the heat transfer, due to the detected role of the air circulation above. For a more realistic case, however, the conjugate heat transfer problem of conduction through the cylinder shell and convection in the fluid phase could be modeled. Ongoing work investigates and modifies more quantities and parameters of the problem, as well as 3D cases where *turbulent* flow exhibits interesting asymmetries in the low filling percentage cases tried.

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