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Coupled Heat and Mass Transfer Processes in Enclosed Environments

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- Belinda Harrison
- Our colleagues

Outline

- Motivation
- Previous work and our contribution
- Definition of some important parameters
- Governing Equations
- Boundary conditions
- Use of COMSOL Multiphysics software package
- Results and Discussion
- Conclusions

Motivation

- An important mode of heat redistribution in very deep mines, caves, human made excavations, and in enclosures surrounded by rockmass
- Plays an important role in cave spelothem's formation and its structural modifications
- An important factor to consider in the modern energy efficient, environmental friendly, and cost efficient homes

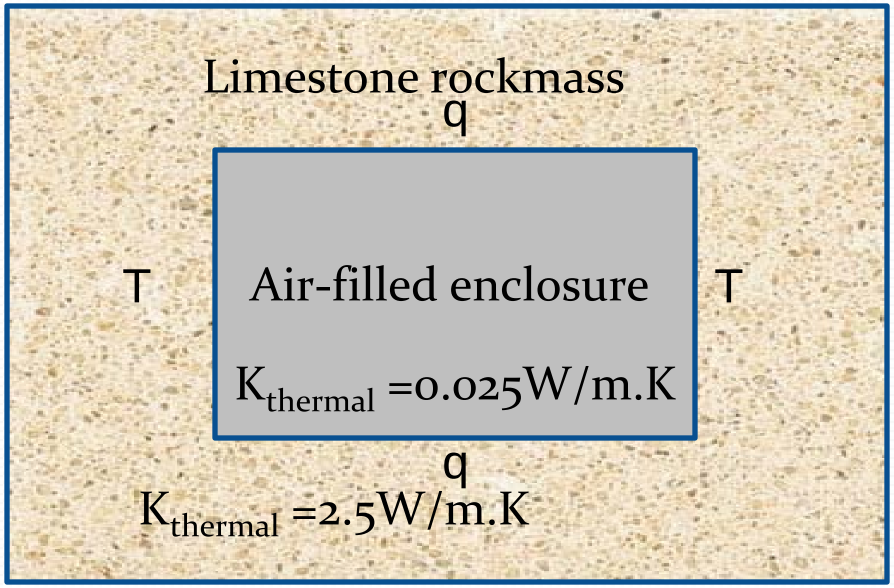
Cave speleothem

(Source: http://www.nps.gov/archive/wica/Speleothems.htm)

Copenhagen office building

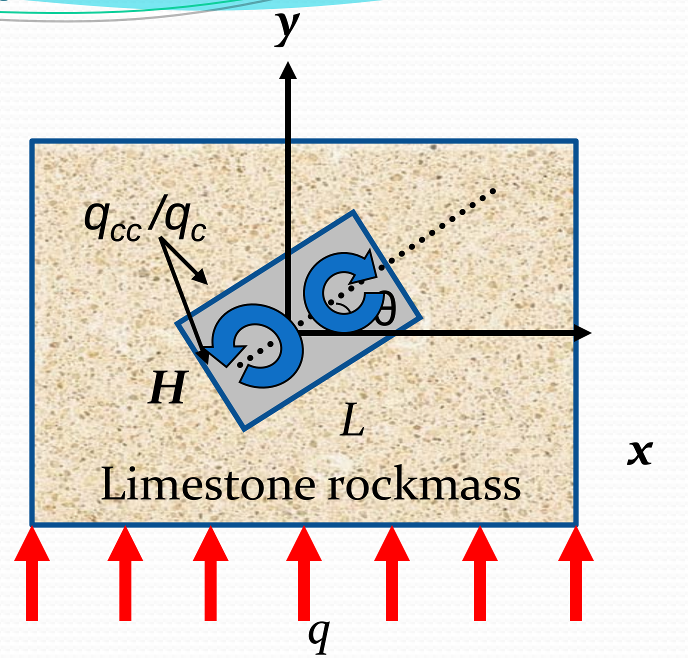
[Glicksman, 2008]

Previous work and our contribution



What are the heat transfer characteristics, measured in terms of heat transfer patterns in the rockmass, and fluid flow behavior and Nu in isolated and buried enclosures, for various enclosure aspect ratios, slopes, and shapes ?

Definition of important parameters



Rayleigh number (Ra)=$g\alpha q H_{eff}^4/\nu\lambda K_r$

Prandtl number (Pr)=ν/λ

Nusselt Number (Nu): q_{cc}/q_c

θ: enclosure slope from horizontal line

q: Applied heat flux

h: Local mesh element size

A=Aspect ratio of an enclosure (L/H)

H_{eff}: Effective length scale [H for θ=0 and otherwise $H\ cos\ (\theta)+L\ sin(\theta)$]

q_{cc}, q_c: Total heat flux passing through any two boundaries of a enclosure when there is convection and conduction heat transfer and when there is only conductive heat transport, respectively

k=Turbulent kinetic energy

ω=Specific dissipation rate of turbulent kinetic energy

K_f/k_r=ratio of thermal conductivities of air and rockmass

Governing Equations

Model 1 | Model 2

Conservation of mass: $\nabla \cdot \vec{u} = 0$

Model 1

Conservation of linear momentum:

$$\vec{u}\cdot\nabla\vec{u} = -\nabla P + (\Pr + F_{lt}\mu_T)\nabla^2\vec{u} + (Ra\Pr T)\hat{j}$$

Transport of k:

$$F_{lt}(\vec{u}\cdot\nabla k = \nabla\cdot[(\Pr + \sigma_k\mu_T)\nabla k] + \frac{\mu_T(\nabla\vec{u})^2}{2} - \beta_k k\omega)$$

Transport of ω:

$$F_{lt}(\vec{u}\cdot\nabla\omega = \nabla\cdot[(\Pr + \sigma_\omega\mu_T)\nabla\omega] + \frac{a\omega\mu_T(\nabla\vec{u})^2}{2k} - \beta\omega^2)$$

Conservation of energy:

$$\frac{K_f}{K_r}\vec{u}\cdot\nabla T = \frac{QH^2}{K_f\Delta T} + \nabla\cdot[(\frac{K_f}{K_r} + F_{lt}K_T)\nabla T]$$

Turbulent thermal conductivity: $K_T = \frac{\mu_T}{\Pr_T}(\frac{K_f}{K_r})$

Model 2

Conservation of linear momentum:

$$\vec{u}\cdot\nabla\vec{u} = -\nabla P + (\frac{\Pr}{\sqrt{Ra}}\frac{K_f}{K_r} + F_{lt}\mu_T)\nabla^2\vec{u} + [\Pr T(\frac{K_f}{K_r})^2]\hat{j}$$

Transport of k:

$$F_{lt}(\vec{u}\cdot\nabla k = \nabla\cdot[(\frac{\Pr}{\sqrt{Ra}}\frac{K_f}{K_r} + \sigma_k\mu_T)\nabla k] + \frac{\mu_T(\nabla\vec{u})^2}{2} - \beta_k k\omega)$$

Transport of ω:

$$F_{lt}(\vec{u}\cdot\nabla\omega = \nabla\cdot[(\frac{\Pr}{\sqrt{Ra}}\frac{K_f}{K_r} + \sigma_\omega\mu_T)\nabla\omega] + \frac{a\omega\mu_T(\nabla\vec{u})^2}{2k} - \beta\omega^2)$$

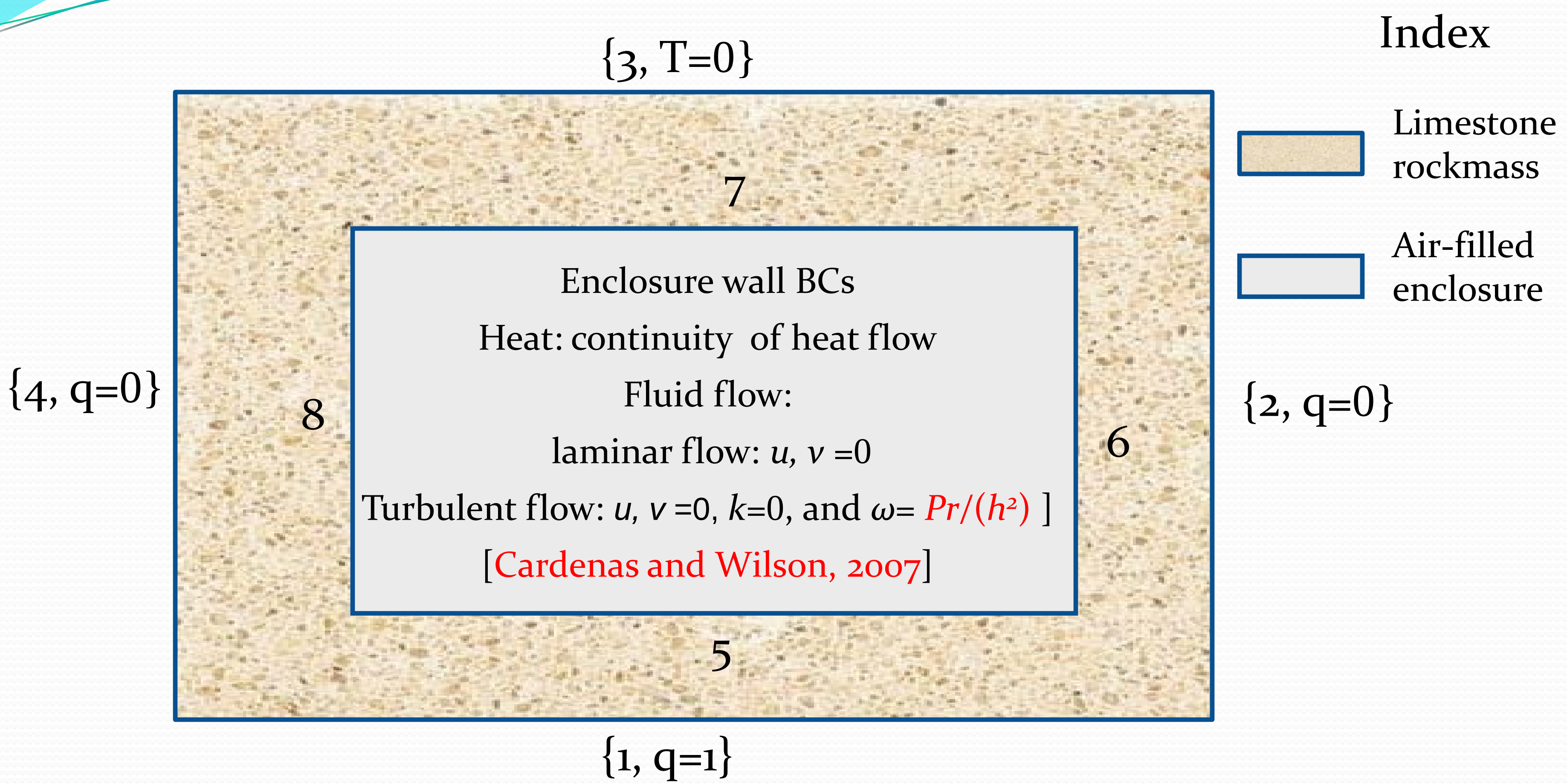
Conservation of energy:

$$\sqrt{Ra}\,\vec{u}\cdot\nabla T = \frac{QH^2}{K_f\Delta T} + \nabla\cdot[(\frac{K_f}{K_r} + F_{lt}K_T)\nabla T]$$

Turbulent thermal conductivity: $K_T = \frac{\mu_T}{\Pr_T}\sqrt{Ra}$

Eddy viscosity: $\mu_T = \frac{k}{\omega}$

Boundary conditions (BCs)



Note: For an infinitely wide enclosure case, periodic boundary conditions for *u, v, p,* and *T* are used for boundaries 6 and 8, which extent to boundaries 2 and 4.

Use of COMSOL Multiphysics software package

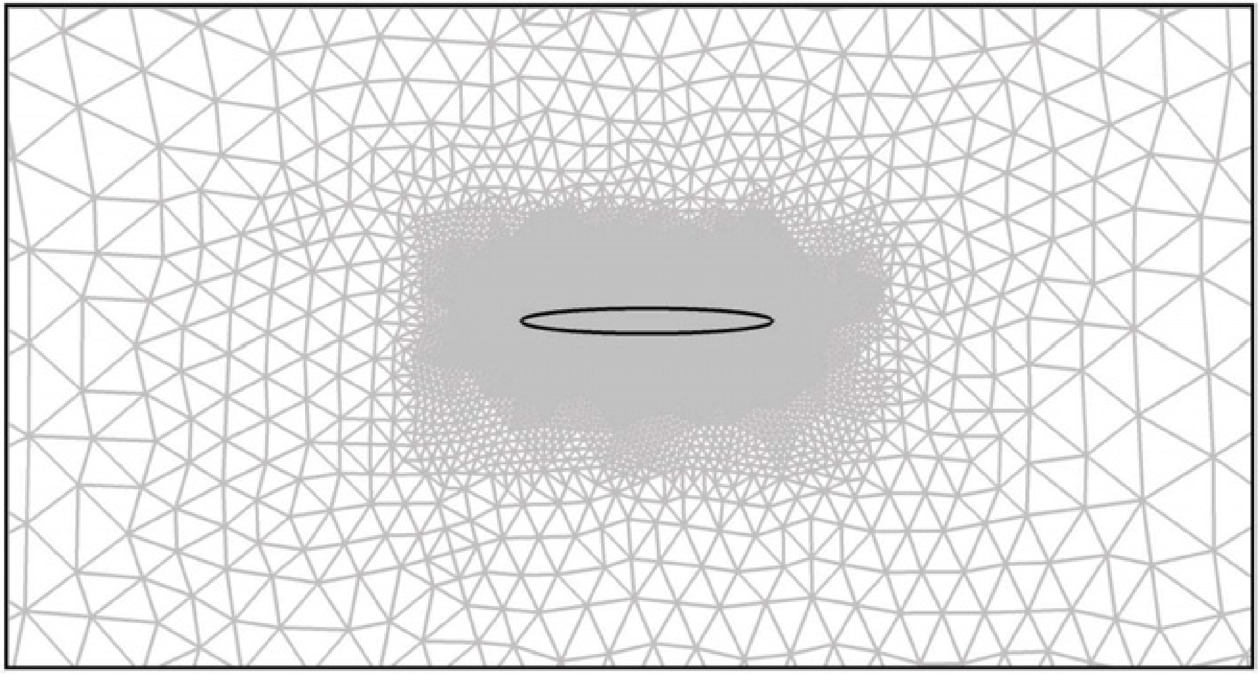
Laminar convection

- Heat transfer
  - Rockmass:
    - Conduction only
  - Enclosure:
    - Conduction and convection
- Momentum transport:
  - Enclosure only
    - Navier-Stokes equation for incompressible fluid

Turbulent convection

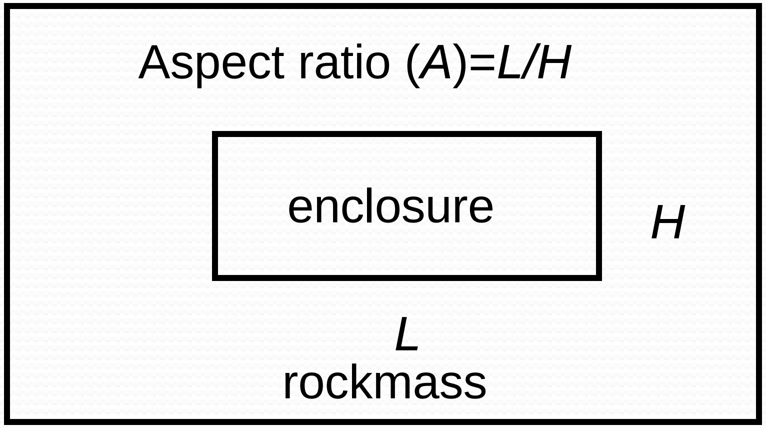
- Heat Transfer
  - Rockmass
    - Conduction only
  - Enclosure :
    - Conduction and convection with turbulent thermal conductivity term
- Momentum transport :
  - Enclosure only
    - RANS k-ω two-equation turbulence model

Result and Discussion

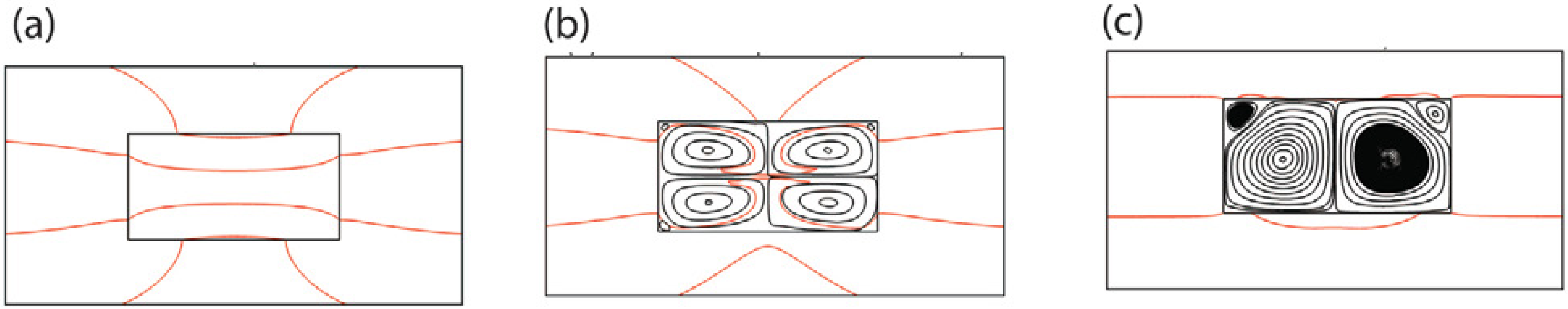


A close-up view of a typical mesh discretization used in the simulations (Note: geometry in black is an elliptical enclosure)

Effect of enclosure aspect ratio

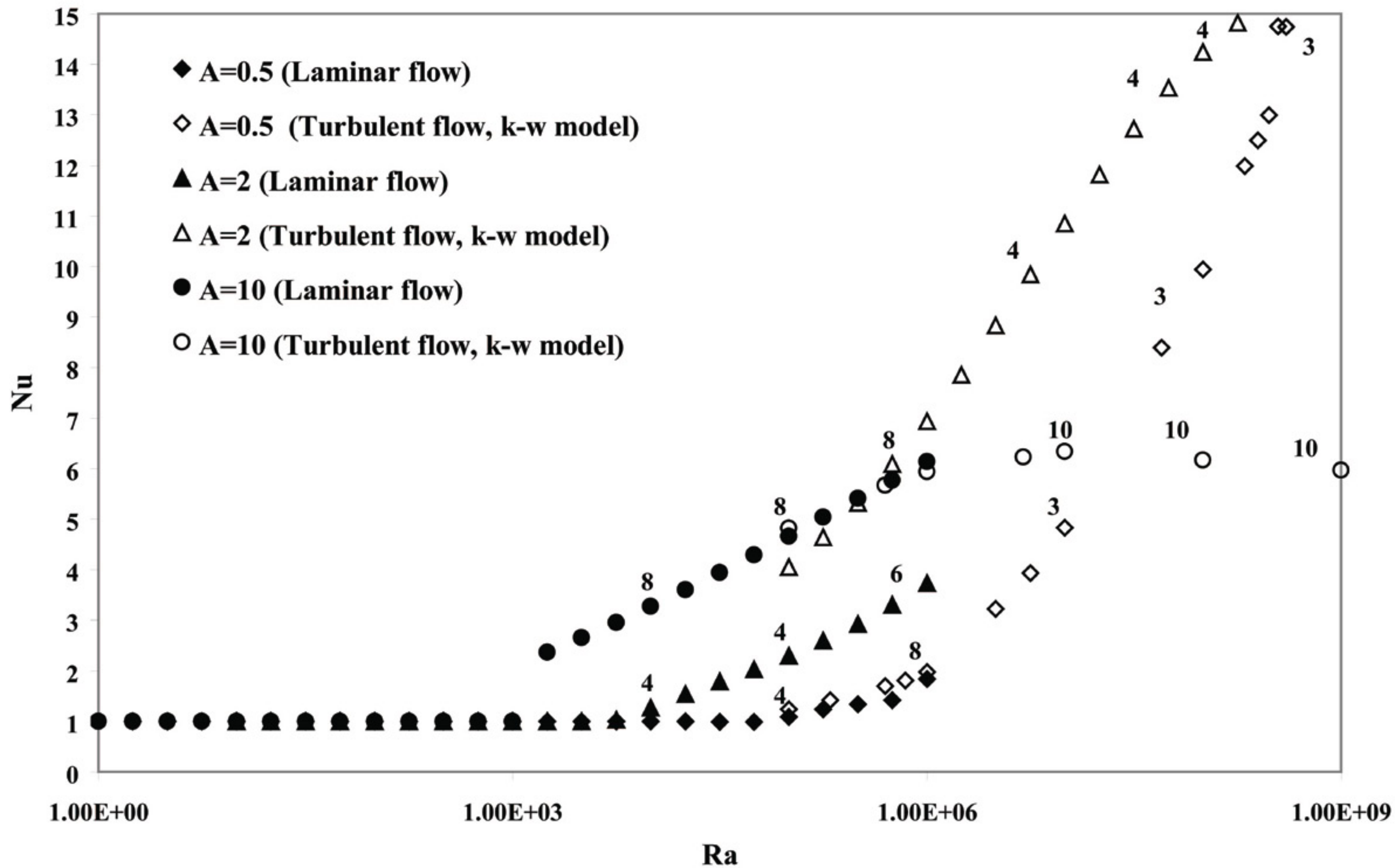


Schematic diagram for the enclosure aspect ratio question

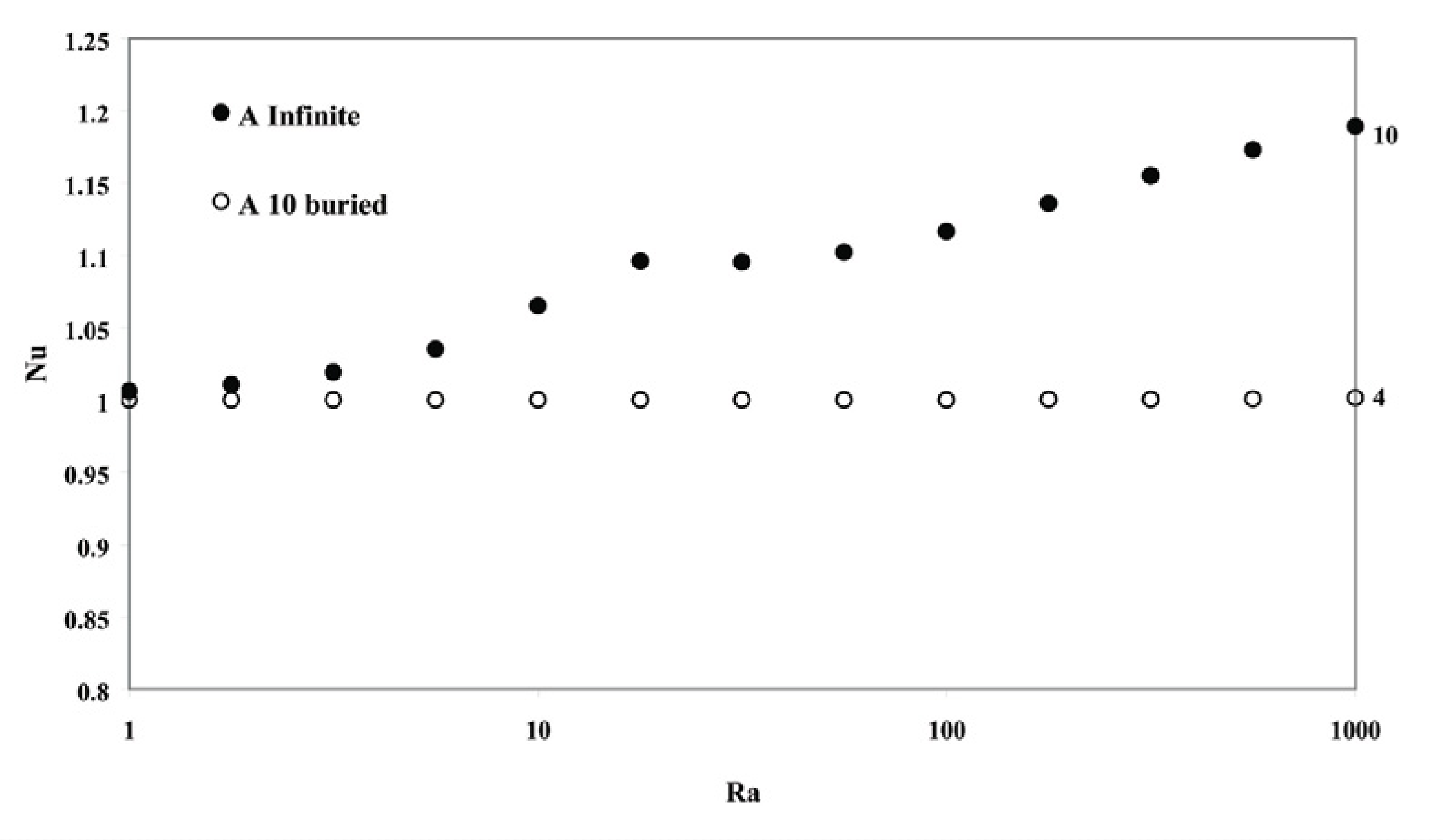


A close up view of the contour plot (in red) of temperature (rockmass and enclosure) and streamline plot (in black) of velocity field in the enclosure subdomain for enclsoure with as A=2, (a) only conductive heat transfer (Ra=0), (b) laminar convection (Ra=1E6), (c) turbulent convection (Ra=1E9) [Note: stacked convection cells in (b) and (c)]

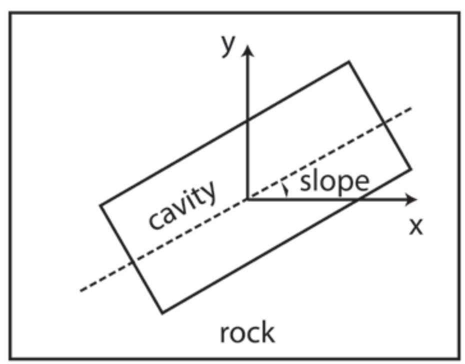
Comparison of *Nu* vs *Ra* behavior for isolated and buried cavities (numbers on the plot show number of convection cells at a particular *Ra*)



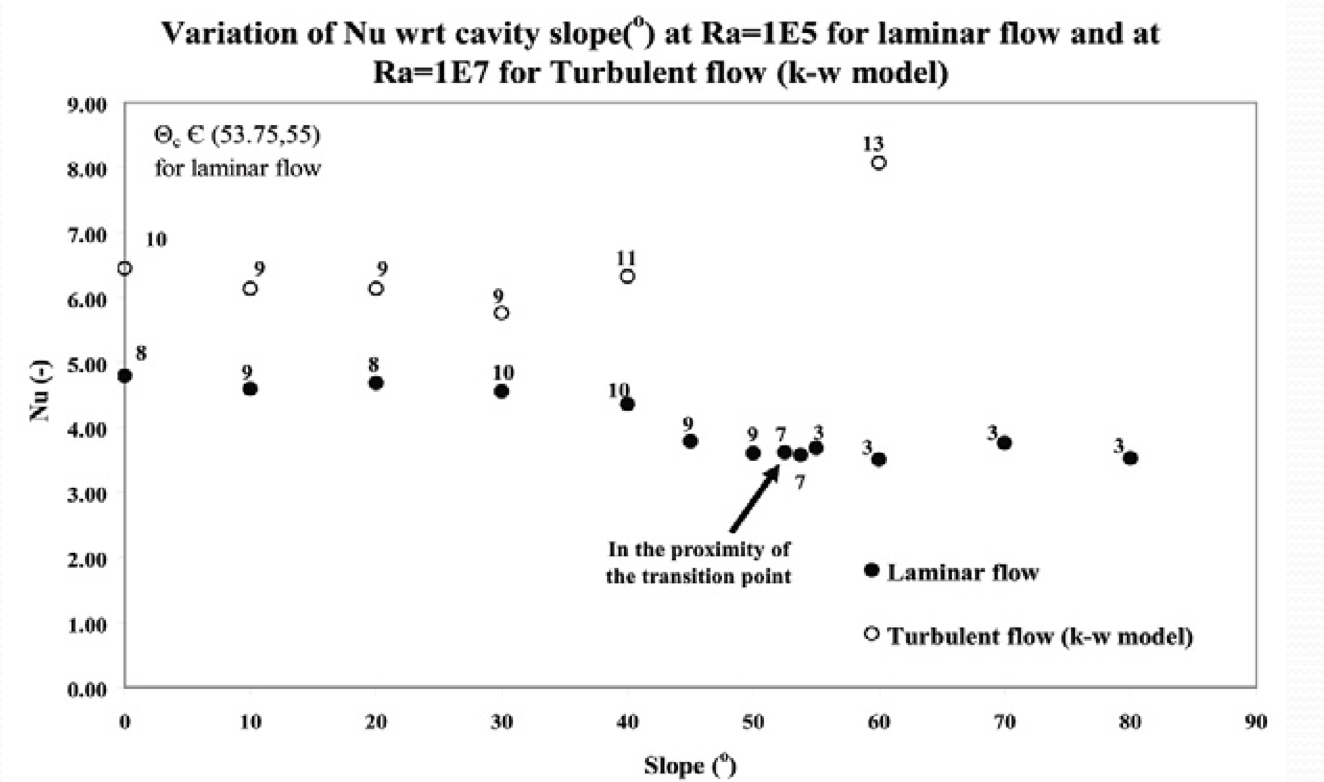
Comparison of *Nu* vs *Ra* behavior between isolated and buried enclosure with A=10 and infinite enclosure aspect ratio (numbers on the plot show number of convection cells at a particular *Ra*)



Effect of enclosure slope



Schematic diagram for the enclosure slope question



Variation of *Nu* wrt enclosure slope at *Ra*=1E5 for laminar flow and at *Ra*=1E7 for turbulent flow (numbers on the plot show number of convection cells at a particular *Ra*)

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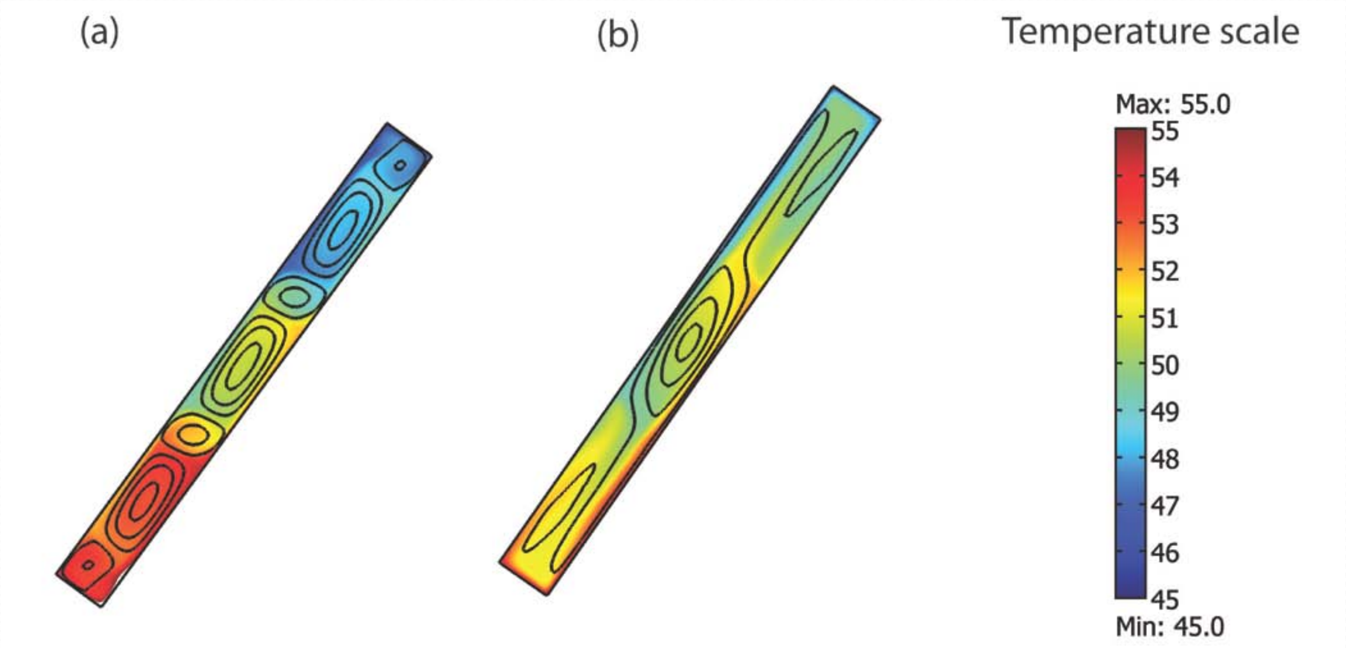
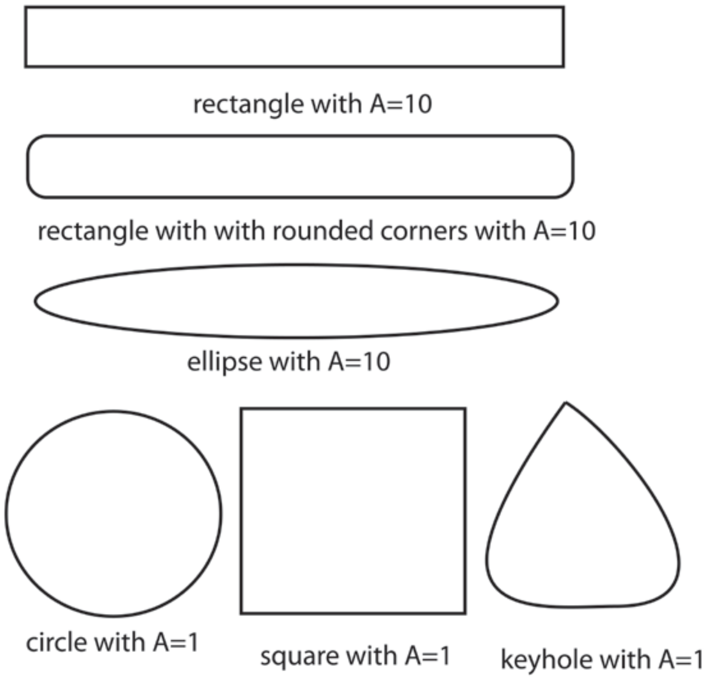


Surface plot of temperature and streamline plot of velocity field in a enclosure with the slope of 53.75º (a) and with the slope of 55º (b) with a laminar fluid flow at Ra=1E5

Effect of enclosure shape

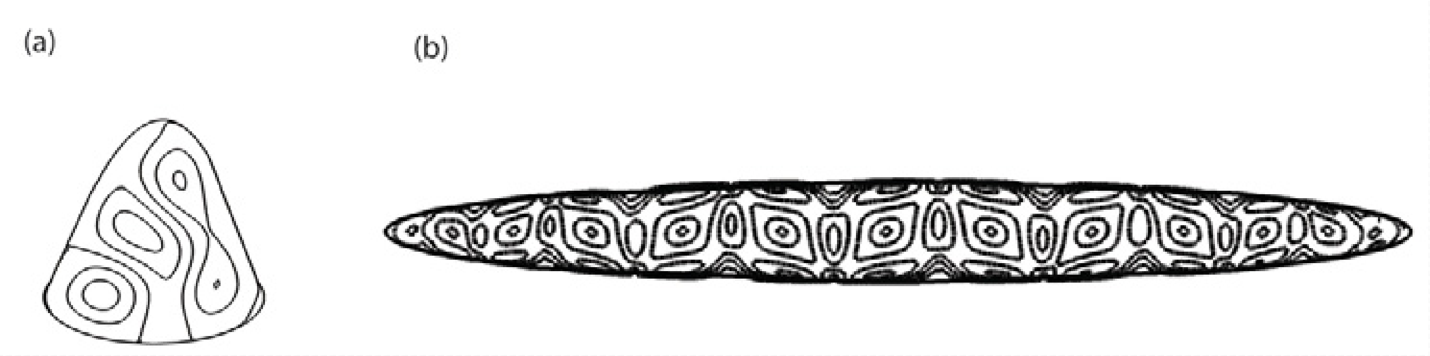


Schematic diagram for the enclosure shape question

Comparison of *Nu* for cavities with various shapes with a laminar flow assumption at *Ra=1E6*

Cavity geometry	A	Nu	# of cells
circle	1	1.31	7
square	1	2.48	4
keyhole	1	5.30	3
rectangle	10	6.41	8
rectangle with rounded corners	10	6.77	9
ellipse	10	5.77	12

...cont'd



Streamline plot of velocity field for a keyhole shaped enclosure (a) and for an elliptical shaped enclosure (b) at *Ra*=1E6 with laminar flow assumption

Conclusions

- Presence of a enclosure, and flow behavior within the enclosure, affect the pattern of heat flux flow in the surrounding rock mass in the proximity of a enclosure.
- Convection cells, almost as if they were intelligent creatures, respond to enclosure aspect ratio, enclosure slope, and enclosure shape alterations by arranging themselves, in such a manner, which optimizes total heat transport from/through that system.
- COMSOL Multiphysics software package is a useful numerical experimenting tool to understand non-linear heat transport processes in enclosed environments. However, for convection-dominated problems (e.g. turbulent convection) convergence remains a challenging issue.

Thank you!

COMSOL support team's response regarding to the convergence issue:

"…You can see that the problem has issues converging over a certain value of the Raleigh number in your problem.

This problem is a hard problem in COMSOL at present and will be handled better in the next version.

I suggest you wait for the next version to tackle this problem. You can find out about the details of the next version at this year's COMSOL conference."

Best regards

COMSOL representative

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Additional material

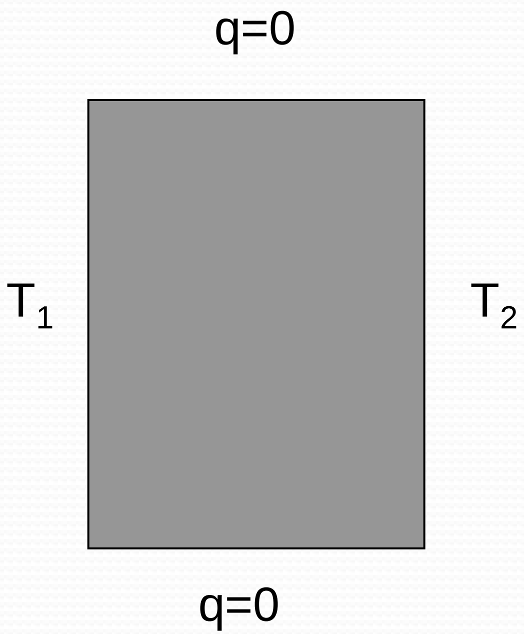
Assumptions

- General
  - Limestone rockmass is homogenous and isotropic
  - Fluid (air) is incompressible
- Air-filled enclosure:
  - Oberbeck-Boussinesq approximation is applicable
  - Steady state for heat transport and fluid flow
  - Laminar flow or turbulent flow (fully turbulent Reynolds averaged flow)
  - k-ω two-equation closure model

Literature review

1 Enclosure heated from sides with constant temperature along vertical boundaries while other boundaries are thermally insulated

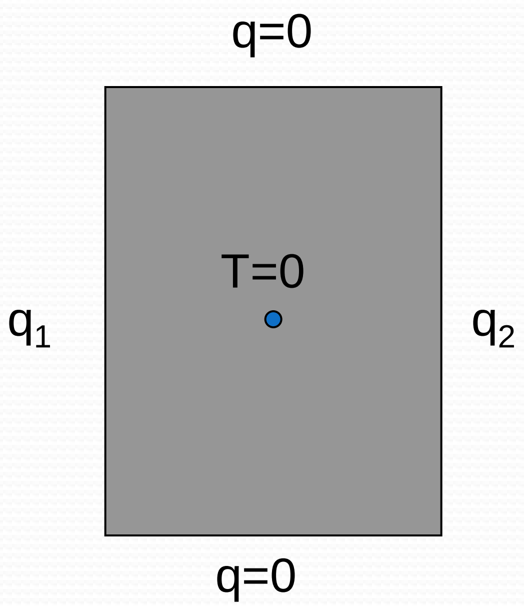
[see Lage and Bejan (1991), Ampofo and Karayiannis (2003), and many others]



$T_1 > T_2$

$q_1 > q_2$

Literature review



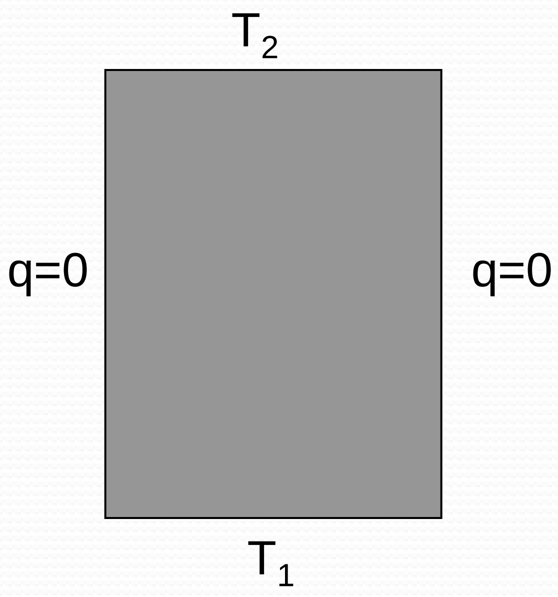
2 Enclosure heated from sides with constant heat flux along vertical boundaries while other boundaries are thermally insulated

[see Kimura and Bejan (1984)]

$T_1 > T_2$

$q_1 > q_2$

Literature review



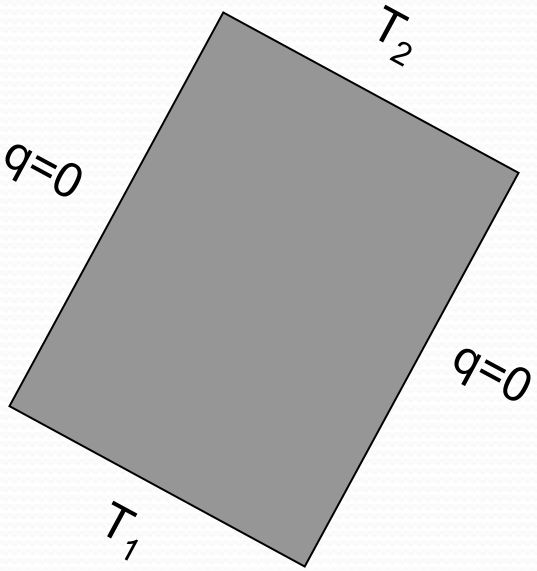
3 Enclosure heated from bottom with constant temperature along horizontal boundaries while other boundaries are thermally insulated

[see Kessler et all. (1984) and Krishnamurti (Part I and Part II,1970)]

$T_1 > T_2$

$q_1 > q_2$

Literature review



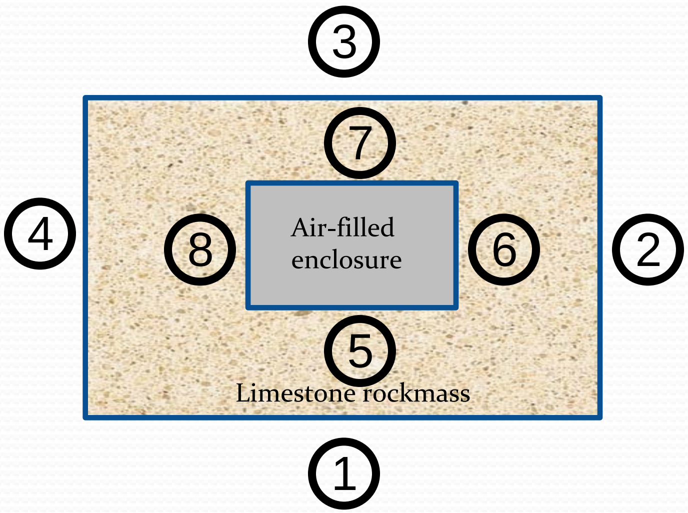
$T_1 > T_2$

$q_1 > q_2$

4 Inclined enclosure with constant temperature along longer boundaries while other boundaries are thermally insulated

[see Bejan (2004)]

Boundary conditions



Boundary #	Flow type			
	Laminar		Turbulent	
	Heat	Fluid	Heat	Fluid
1	q=1	[-]	q=1	[-]
2,4	q=0	[-]	q=0	[-]
3	T=0	[-]	T=0	[-]
5,6*,7,8*	continuity	No-slip	continuity	No-slip, k=0, and $\omega=Pr/h^2$

Note: For an infinitely wide enclosure case, periodic boundary conditions for u, v, p, and T are used for boundaries 6 and 8, which extent to boundaries 2 and 4