Effects of Shear-thinning and Elasticity in Flow around a Sphere in a Cylindrical Tube

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Introduction

Flow around a sphere in cylindrical tubes filled with both purely viscous and viscoelastic liquids is of practical and fundamental interest.

- Fixed or fluidized bed
- Falling ball viscometry
- Emulsion or suspension processing
- Filled polymer melts processing
- Sphere sedimentation in viscoelastic fluids is a benchmark problem in the computational rheological community

Governing equations and boundary conditions

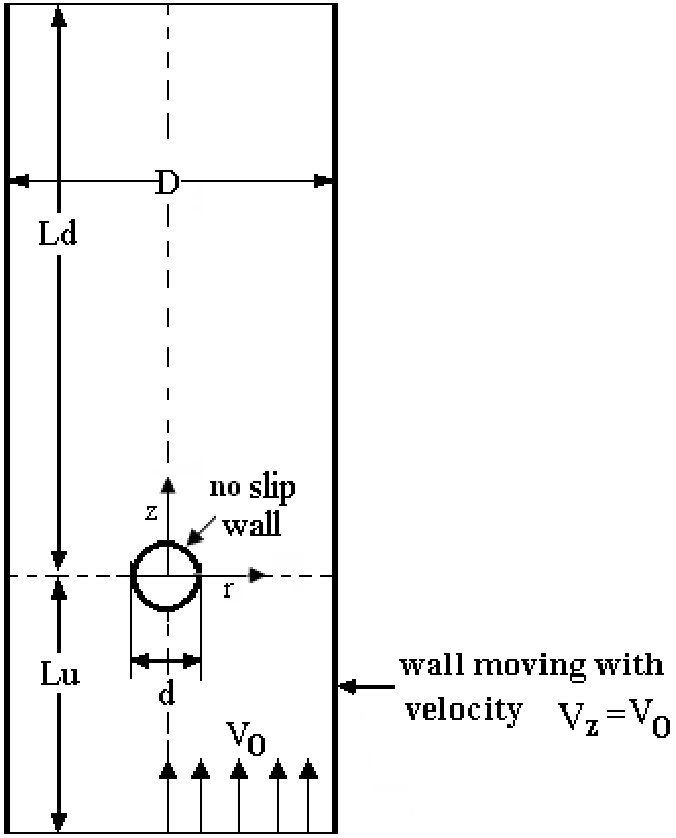


Figure 1. Schematic diagram of flow around a sphere in a tube

(Lu=10R, Ld=30R, d/D=0.5)

Governing equations

Continuity equation

$$\nabla \bullet U = 0$$

Momentum equation

$$-\nabla p + \nabla \bullet \sigma = 0$$

Constitutive equation

Newtonian

$$\sigma = \eta_0 \dot{\gamma}$$

Constant shear viscosity w/o elasticity

Non-Newtonian

Inelastic

$$\sigma = \eta_c\left(\left|\dot{\gamma}\right|\right)\dot{\gamma}$$

$$\eta_c\left(\left|\dot{\gamma}\right|\right) = \eta_0\left[1 + \left(\lambda_c\left|\dot{\gamma}\right|\right)^2\right]^{(n-1)/2} (n < 1)$$

Carreau model (shear-thinning w/o elasticity)

Viscoelastic

Oldroyd-B

Constant shear viscosity with elasticity

Phan-Thien-Tanner (PTT) model

Shear-thinning with elasticity

U, velocity vector; σ, extra stress tensor; p, pressure

η_0, zero-shear-rate viscosity; shear rate, $\dot{\gamma} = \nabla U + (\nabla U)^T, \left|\dot{\gamma}\right| = \sqrt{\frac{1}{2}\dot{\gamma} : \dot{\gamma}}$

Oldroyd-B

$$\sigma + \lambda_1 \overset{\nabla}{\sigma} = \eta_0 \left(\dot{\gamma} + \lambda_2 \overset{\nabla}{\dot{\gamma}} \right)$$

λ_1 and λ_2 are the relaxation and retardation times, respectively.

$$\overset{\nabla}{\sigma} = \frac{\partial \sigma}{\partial t} + (u \cdot \nabla)\sigma - \left(\nabla u^T \cdot \sigma + \sigma \cdot \nabla u \right)$$

Momentum equation $-\nabla p + \nabla \bullet \sigma = 0$ No diffusivity term

Trick: Elastic Viscous Split Stress (EVSS)

$$\sigma = \eta_N \dot{\gamma} + \tau$$

New momentum equation $-\nabla p + \nabla \cdot \left(\eta_N \dot{\gamma} \right) + \nabla \cdot \tau = 0$ with diffusivity term

$$\tau + \lambda_1 \overset{\nabla}{\tau} = \eta_E \dot{\gamma}$$

$$\eta_0 = \eta_N + \eta_E, \; s = \eta_N / \eta_0 = \lambda_2 / \lambda_1$$

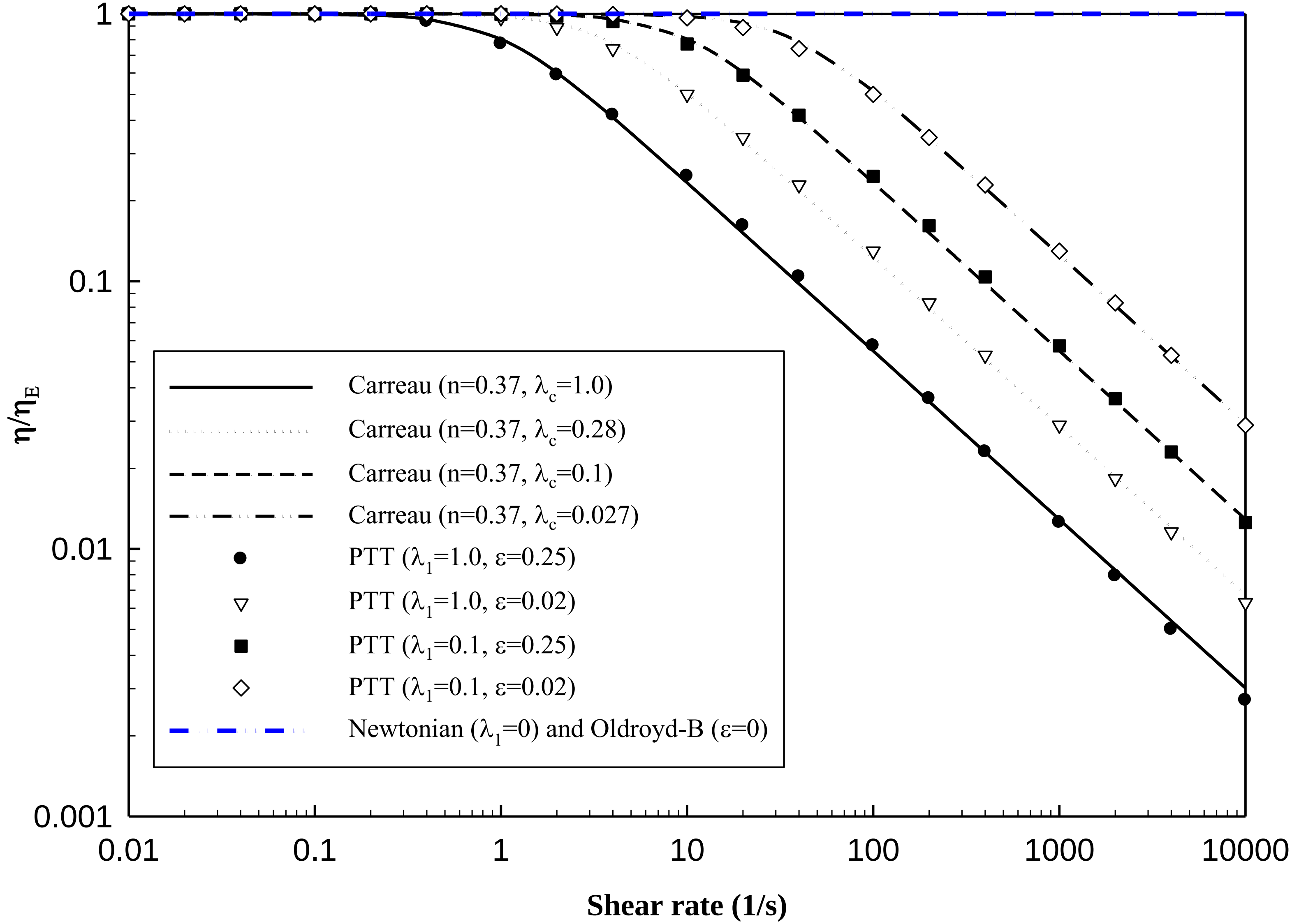
Phan-Thien-Tanner (PTT) model

$$\tau + \lambda_1 \overset{\nabla}{\tau} + \varepsilon \frac{\lambda_1}{\eta_E} tr(\tau)\tau = \eta_E \dot{\gamma}$$

ε, extensibility parameter, determining the shear-thinning behavior & extensional viscosity

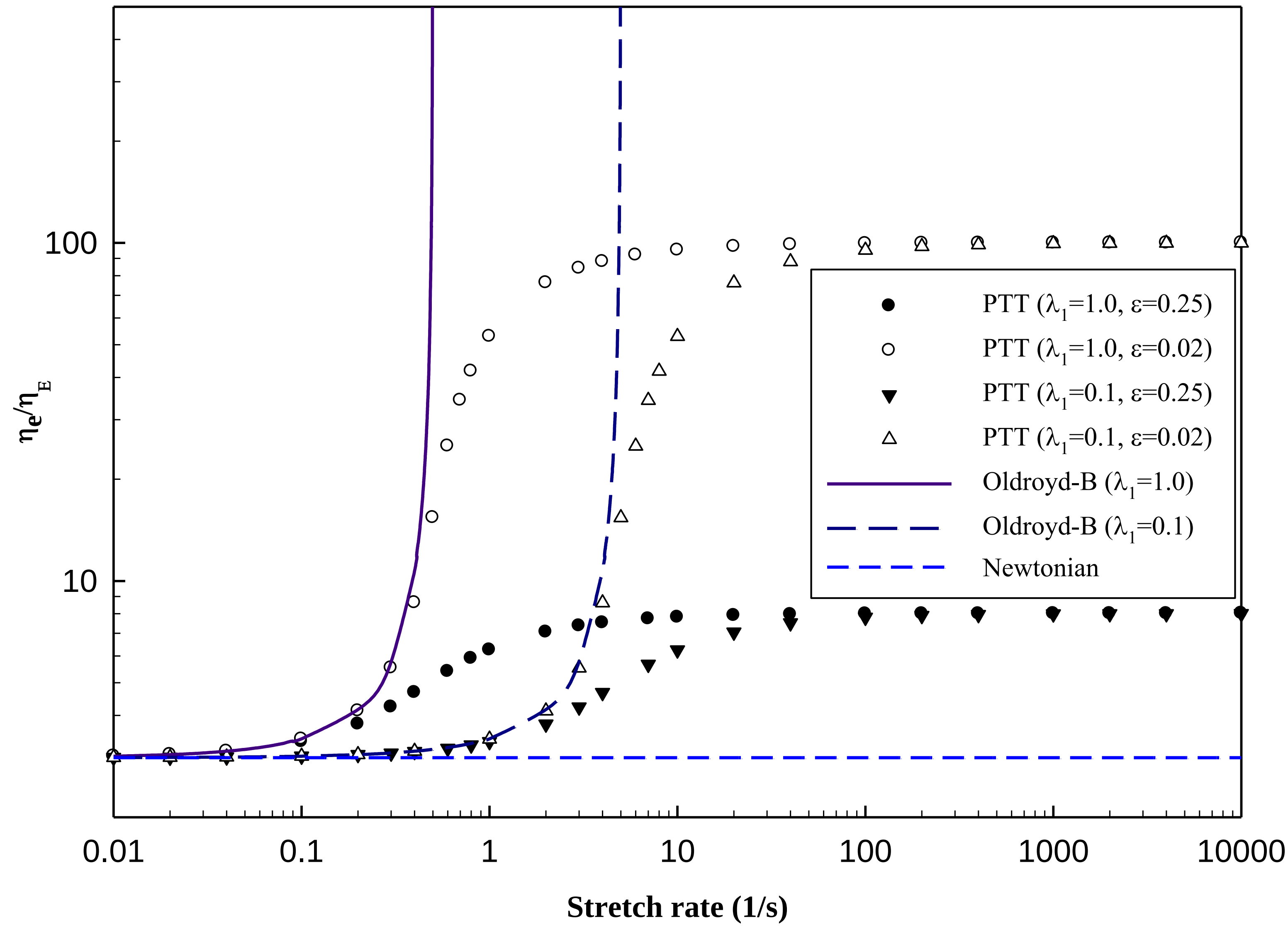
ε=0.02 -> polymer solutions

ε=0.25 -> polymer melts



PTT model, at the same λ_1, a larger ε predicts a lower shear viscosity

Figure 2. Shear viscosity vs. shear rate



PTT model, at the same λ_1, a larger ε predicts a lower extensional viscosity

Figure 3. Extensional viscosity vs. stretch rate

Boundary conditions

Inlet	$u_r = 0, u_z = V_0, \tau_{rr} = \tau_{rz} = \tau_{\theta\theta} = \tau_{zz} = 0$
On the tube wall	$u_r = 0, u_z = V_0$
On the sphere surface	$u_r = u_z = 0$
Symmetry	$u_r = 0, \tau_{rz} = 0$
Exit	Pressure =0, no viscous stress

Dimensionless number: Deborah number
(De, a measure of elasticity)

$$De = \lambda_1 \frac{\bar{u}}{R}$$

R, radius of the sphere; $\bar{u}$ **, mean velocity**

Drag force $$F = \int_{\partial\Omega} \left[\left(-pI + \sigma\right).n\right].e_z \, ds$$

Drag coefficient $$K = \frac{F}{6\pi\eta_0 \bar{u} R}$$

Simulation procedure

1. COMSOL Multiphyscis (3.5a)
2. Quadrilateral elements
3. Element choices:

   Velocity-pressure coupling, Lagrange-$P_2P_{1;}$

   Stresses, Lagrange-Linear
4. For Carreau fluid flow, Non-Newtonian Flow Module
5. For viscoelastic fluids flow, combination of Incompressible Navier-Stokes Module (momentum equation and continuity equation) and PDE mode (constitutive equation)
6. Direct UMFPACK and Parametric Solver

Results and discussion

Validation

Table 1. Comparison of drag coefficient (Oldroyd-B, s=0.5)

De	Present	Lunsmann et al. (1993)
0.0	5.94739	5.94716
0.3	5.69385	5.69368
0.6	5.41221	5.41225
0.9	5.25654	5.25717
1.2	5.18493	5.18648
1.5	5.16132	5.15293

Validation

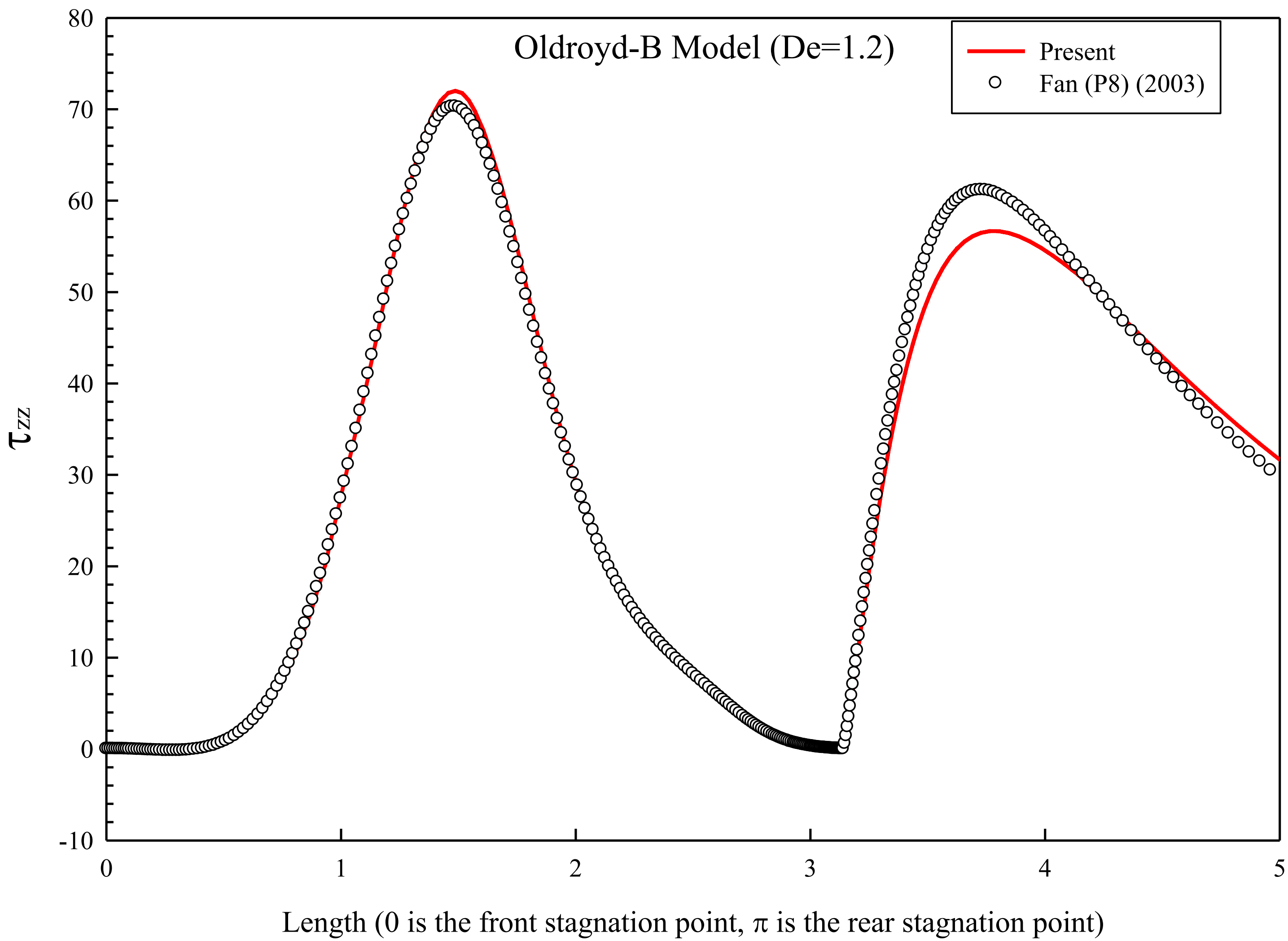


Figure 4. Comparison of τ_{zz} along the sphere surface and in the downstream center line

Effects of shear-thinning and elasticity on drag coefficient

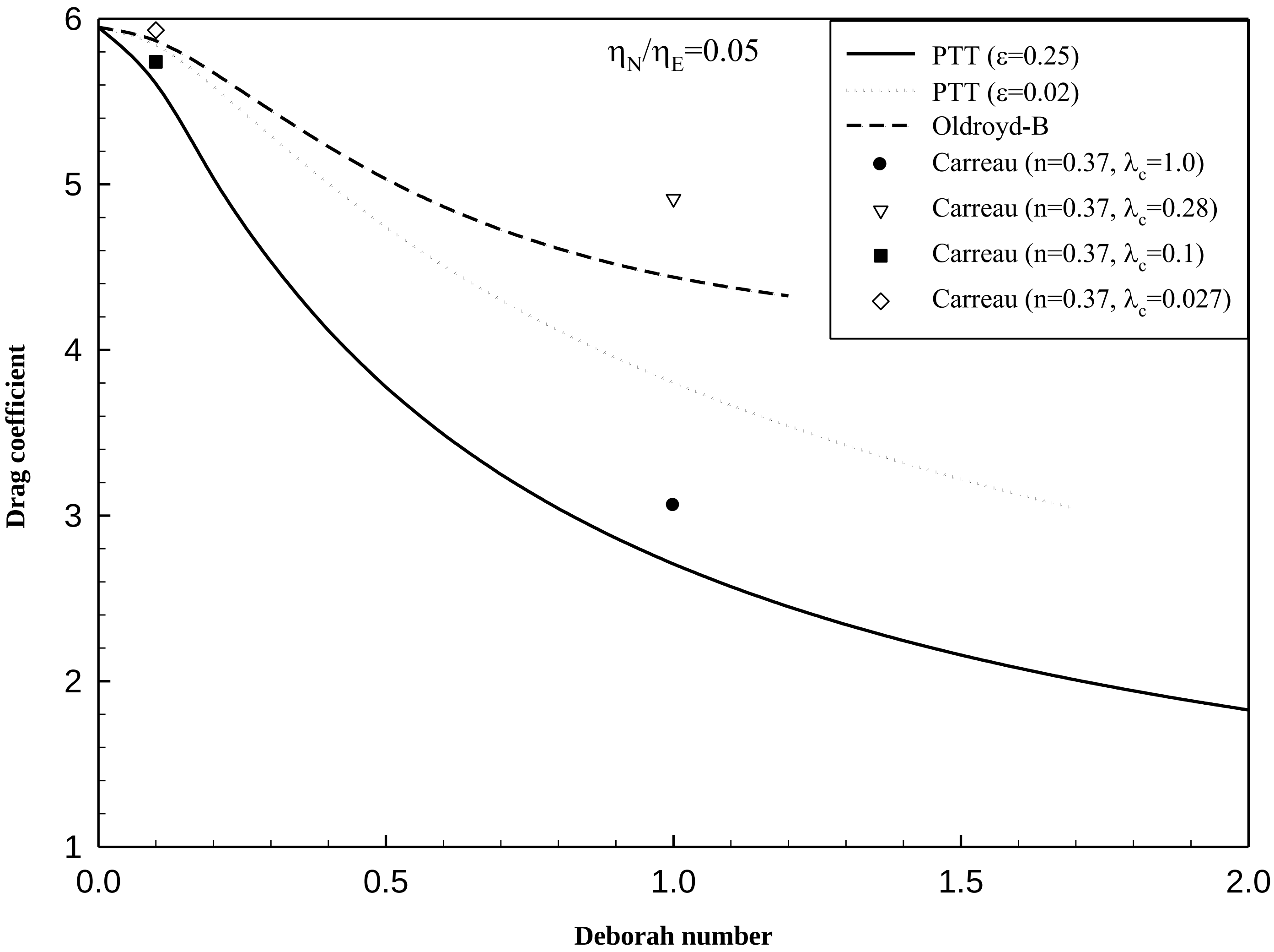


Figure 5. Drag coefficients at different constitutive equations

Effects of shear-thinning and elasticity on velocity overshoot

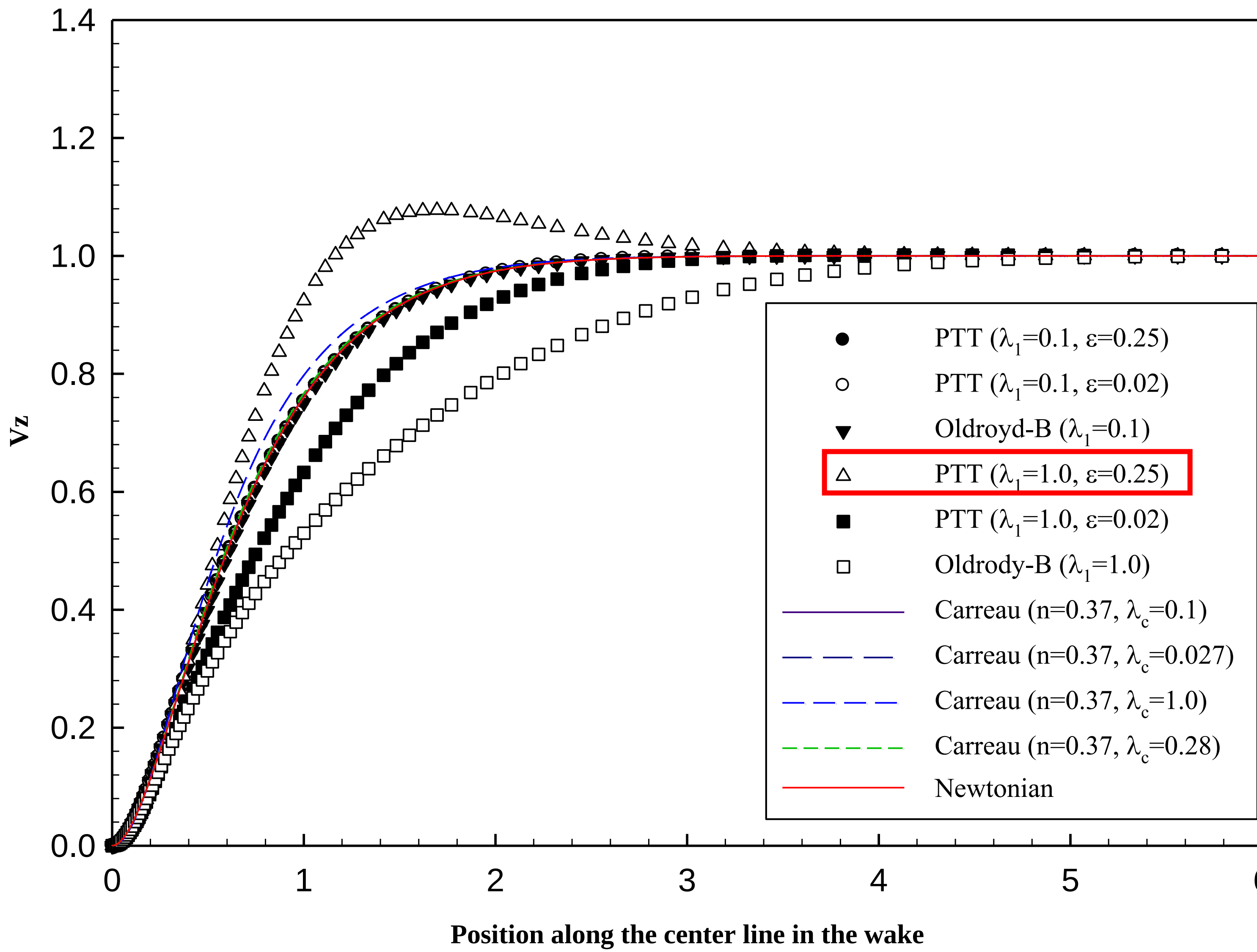


Figure 7a. Velocity at the downstream center line with $\lambda_1 \leq 1.0$

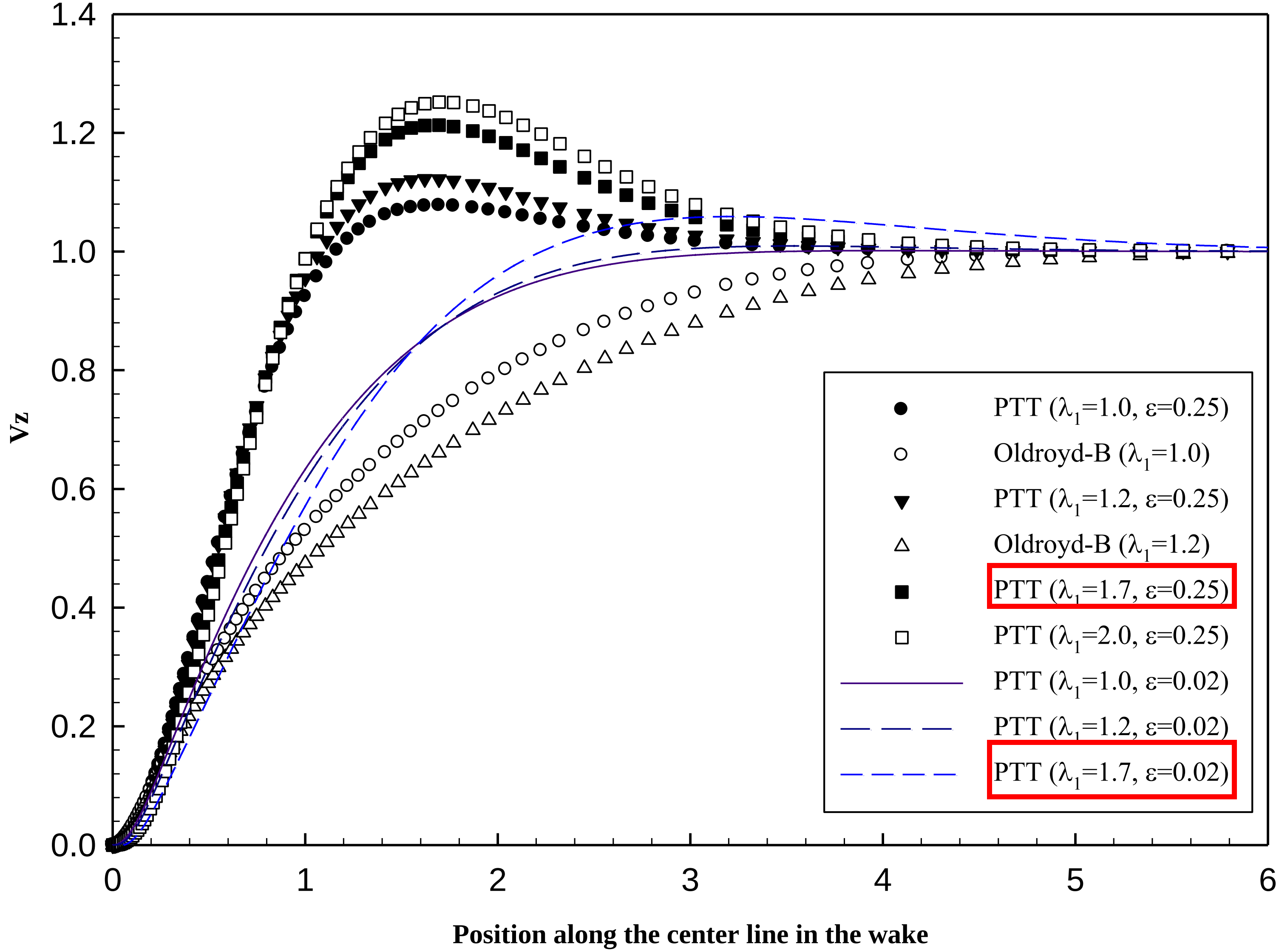


Figure 7b. Velocity at the downstream center line with $\lambda_1 \geq 1.0$

Conclusions

- Both elasticity and shear-thinning lead to a reduction in drag coefficient
- Neither elasticity nor shear-thinning alone gives rise to velocity overshoot at the downstream center line
- Velocity overshoot should be attributed to the synergistic effect of shear-thinning and elasticity behaviors

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