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micro and nanoelectronics
microsystems
ambient intelligence
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COMSOL Assistance for the Determination of Pressure Drop in Complex Microfluidic Channels

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Introduction

- Microsystems for Biotechnology require an accurate determination of pressure drops

→ for flows in microfluidic networks

→ for maximum pressure determination (fragile sealed cover)

→ for two-phase microflows

- Laminar pressure drop $\Delta P \sim (\mu L / d^4) Q$

→ ΔP is very sensitive to the characteristic cross-dimension d

→ Luckily Q and L are small

- There is a requirement for precise 3D pressure drop calculation

- This work addresses

(1) Rectangular (etched) channels

(2) Pillared channels

(3) Non-Newtonian, visco-elastic fluids (whole blood, alginate)

1. Rectangular microchannels – general approach

- Approached analytical expressions (based on a nearly bi-quadratic profile)

$$\Delta P = R\,Q$$

$$R = 4\mu L \big/ [w\,d\,\min(w^2,d^2)\,q(\varepsilon)]$$

$$q(\varepsilon) = 1/3 - \left(64/\pi^5\right)\varepsilon\tanh\left(\pi/2\,\varepsilon\right)$$

$$\varepsilon = \min(w/d, d/w)$$

Expressions for different cross sections Bruus (Theoretical Microfluidics, 2009),
Berthier (Microfluidics for Biotechnology, 2010)

- Numerical approach

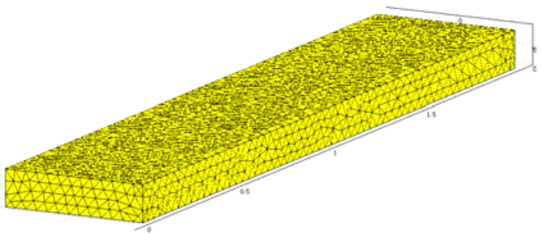
(1) if aspect ratio < 1, 2D-Helle-Shaw calculation

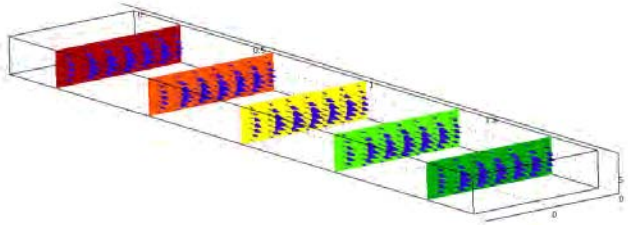
$$\rho\left(\vec{U}.\nabla\right)\vec{U} = -\nabla P + \mu\,\Delta\vec{U} - 12\mu/d^2\,\vec{U}$$

(2) else full 3D

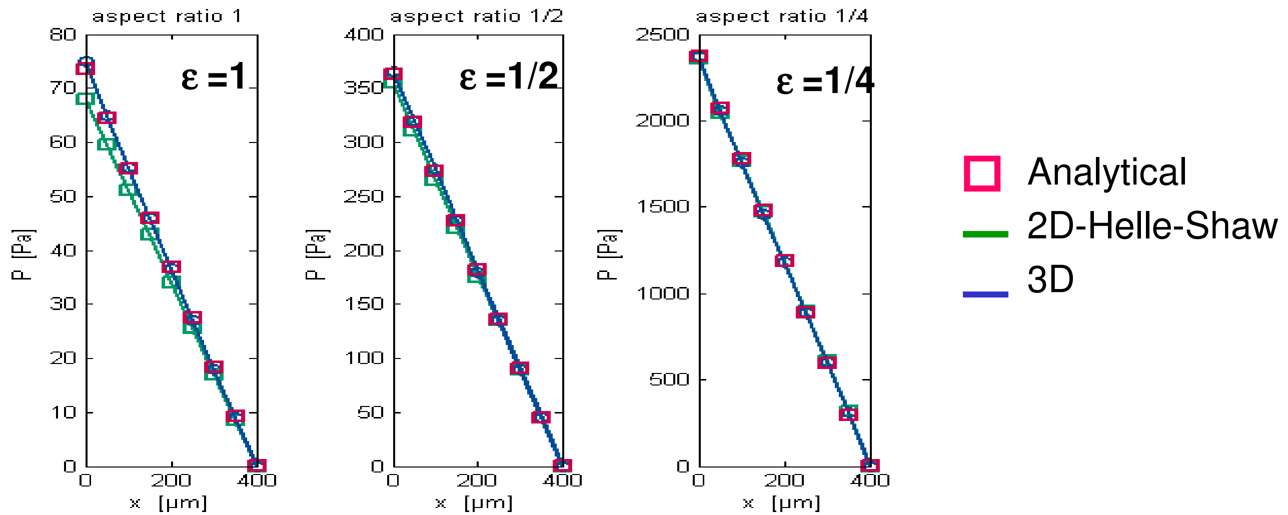
1. Rectangular microchannels – calculation results

- Meshing must be sufficiently small (> 4 meshes in the smaller dimension)



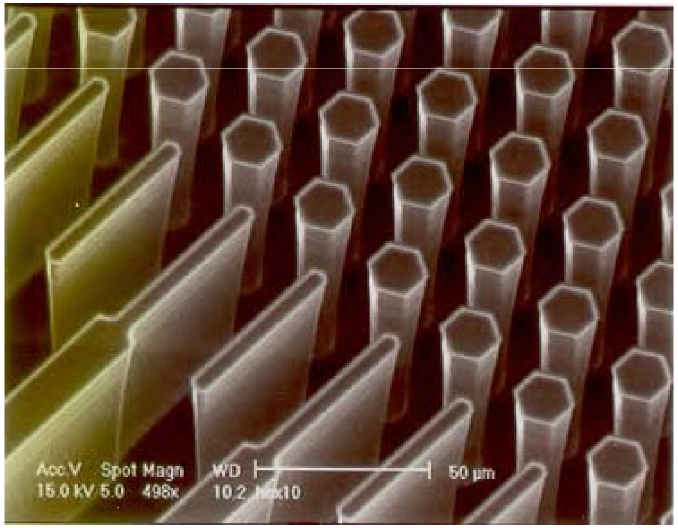


- For aspect ratios ε < ½ , good agreement between the three approaches.

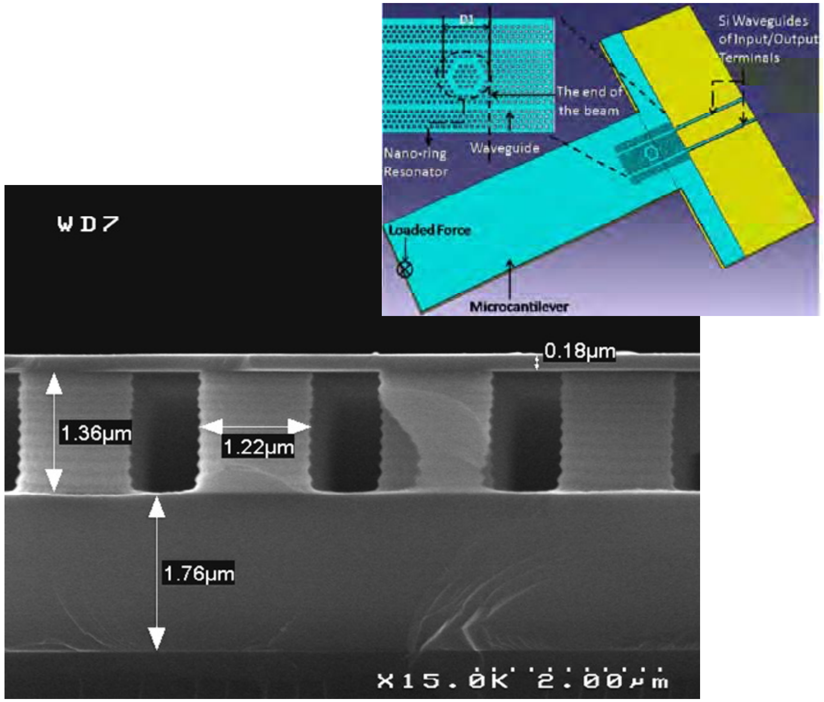


2. Pillared microchannels - Examples

- Pillars are used for capturing biologic targets (DNA, proteins, cells)
  (pillar aspect ratio h/d > 1)
- Pillar are used for reinforcing cover sealing (direct bonding)
  (often pillar aspect ratio h/d < 1)

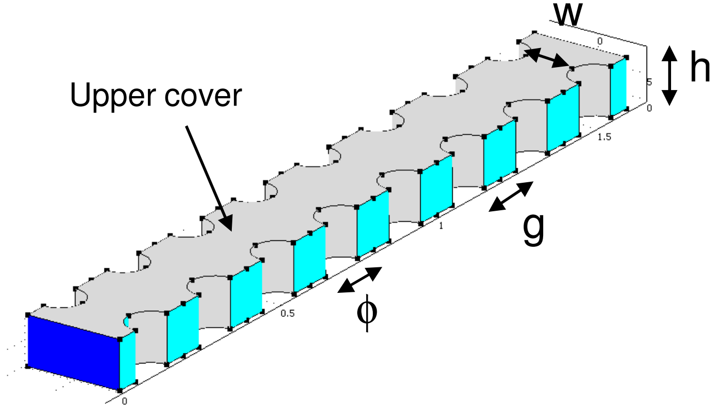


Pillars in a proteomic reactor

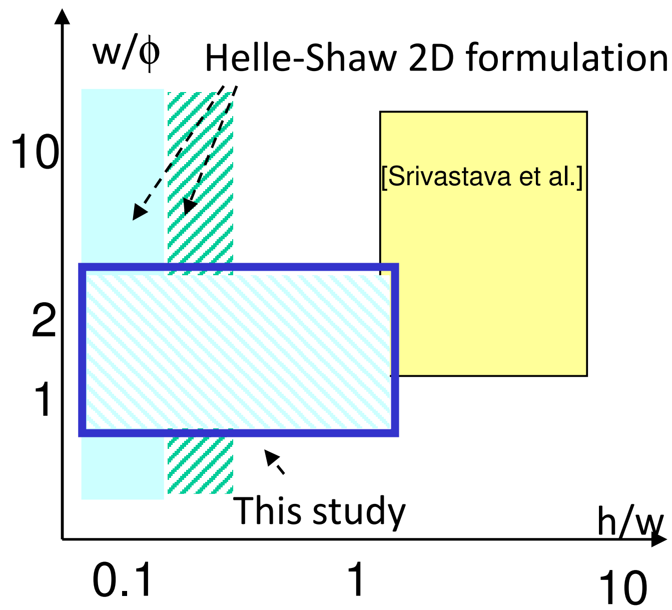


Assistance for sealing microsystem cover plates
In a microfluidic resonator

2. Pillared microchannel - Validity domain



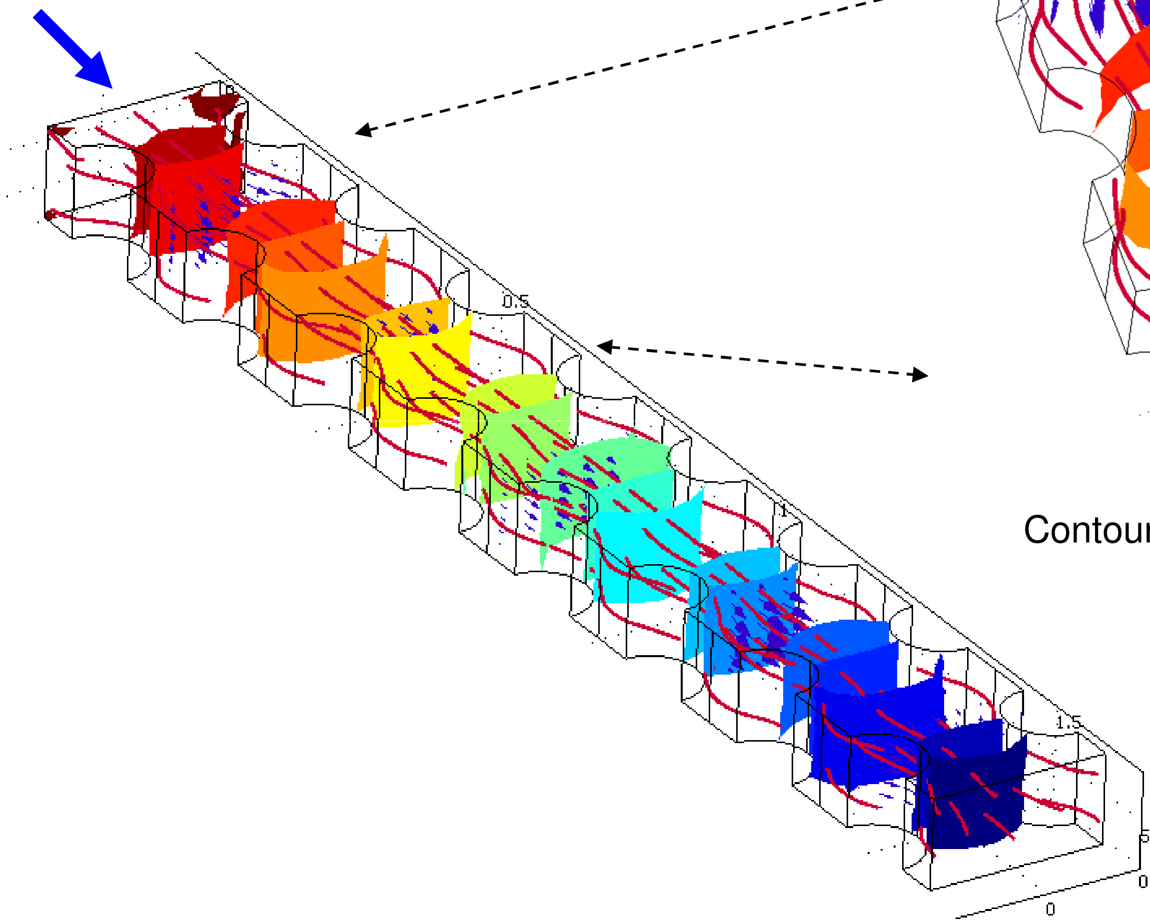
$$\Omega = \left\{ \begin{array}{c} h/w \le 1 \\ 0.5 \le g/w \le 2 \\ 0.5 < w/\phi < 2 \\ w << W \\ g << L \end{array} \right\}$$

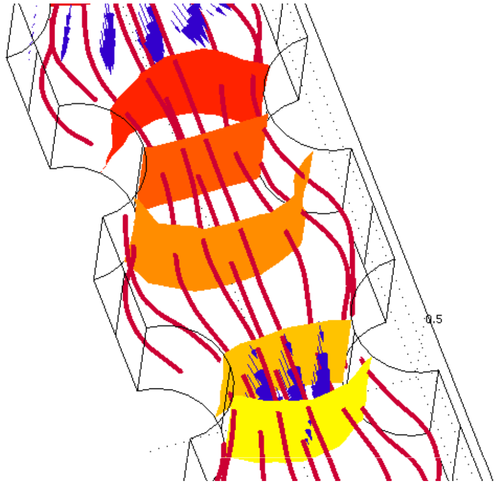


N. Srivastava, C. Din, A. Judson, N.C. MacDonald, C.D. Meinhart,
*A unified scaling model for flow through a lattice of microfabricated posts, Lab Chip, **10**, 1148-1152, 2010.*

2. Pillared microchannel – Pressure/flow field

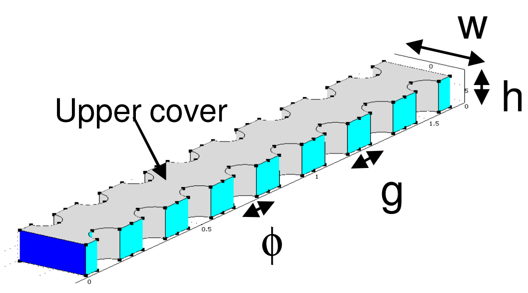
Re < 0.1

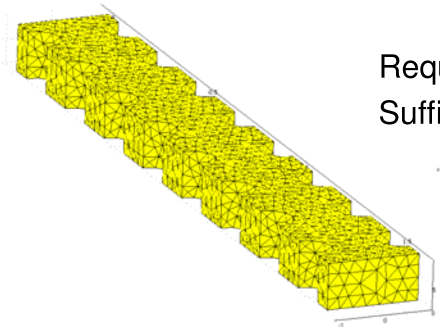




Contour plot of the pressure field

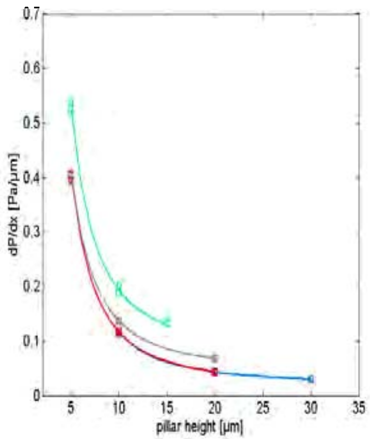
2. Pillared microchannel – Results -1





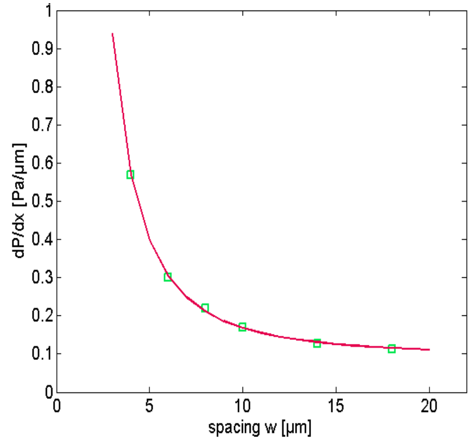
Requirement:
Sufficient number of meshes

w = 10, 20 µm
g = 5, 10 µm



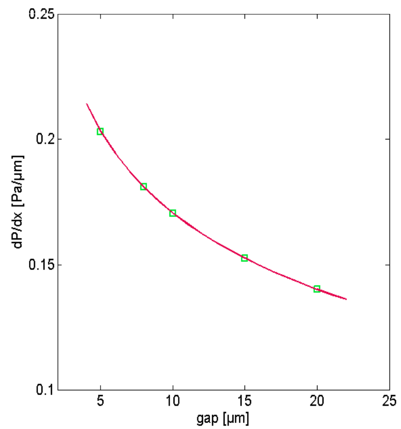
$\delta P/\delta x \sim 1/h^2$

h = 10 µm
g =10 µm
ϕ =10 µm



$\delta P/\delta x \sim 1/w^2$

h = 10 µm
ϕ =10 µm



$\delta P/\delta x \sim 1/g^{0.1}$

2. Pillared microchannel – Results-2

Model →

- $\delta P/\delta x \sim 1/h^2$
- $\delta P/\delta x \sim 1/w^2$
- $\delta P/\delta x \sim 1/g^{0.1}$

→

$$\frac{\partial P}{\partial x} \propto \frac{\mu Q}{h^2 w^2 g^{0.1}}$$

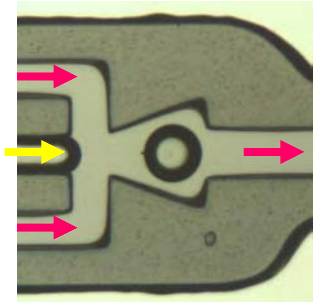
Dimensional analysis $P\,(h, w, g, \phi, x)$ →

$$\frac{\partial P}{\partial x} \approx \frac{\mu Q}{h^2 w^2} \frac{\phi^{0.1}}{g^{0.1}}$$

Correlation →

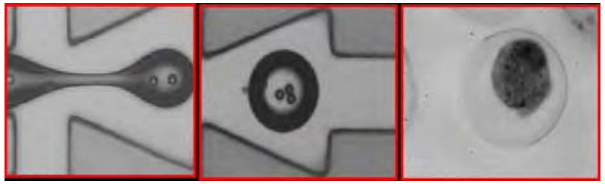
$$\frac{\partial P}{\partial x} \approx \frac{130 \mu Q}{h^2 w^2} \frac{\phi^{0.1}}{g^{0.1}}$$

3. Non-Newtonian fluids - Introduction



- In Biotechnology, non-diluted fluids are increasingly used
- Examples : whole blood for cell/plasma separation

  alginates for cell encapsulation



- Non-Newtonian visco-elastic fluids
- Shear-thinning fluids: viscosity decreases with the shear
- 'power' fluids or 'Ostwald' fluids obey the law

$$\eta = K\,\dot{\gamma}^{n-1}$$

- Rabinowitsh-Mooney law for cylindrical ducts

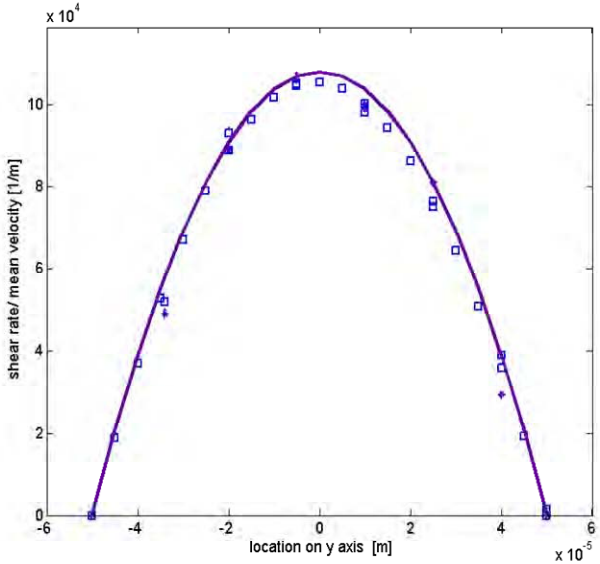
$$\Delta P_{RM} = \frac{2^{(n+2)}\,L\,K}{R}\left(\frac{3n+1}{n}\right)^{n}\left(\frac{\overline{U}}{R}\right)^{n}$$

- Kozicki, Muzychka, Miller approximations for rectangular ducts

$$\Delta P = \frac{2^{3n+2}\,K\,L}{w^{n+1}}\left(\frac{c_1}{n} + c_2\right)^{n}\overline{U}^{\,n}$$

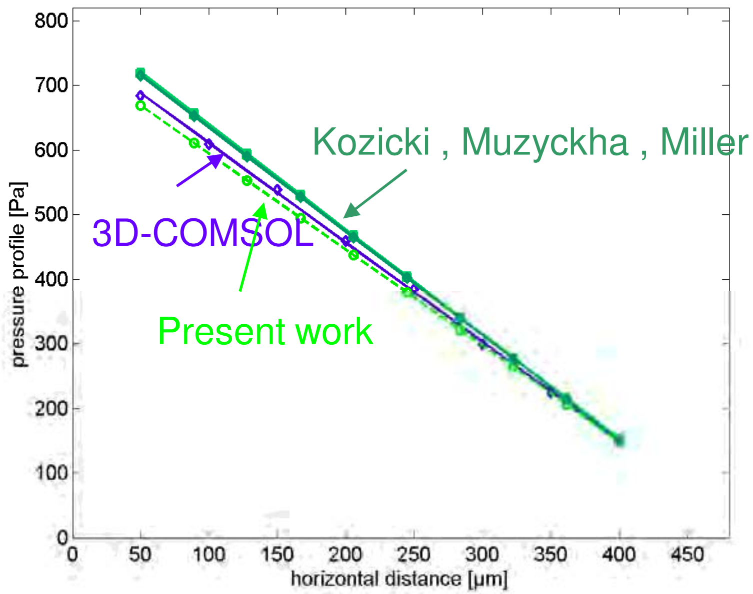
3. Non-Newtonian fluids – Results for a square channel

Wall shear rate (3D-COMSOL)



$$\dot{\gamma}(y) = 1.2\,\overline{U}\left[\frac{k+1}{k}\right]^2 \frac{k}{a}\left[1-\left(\frac{y}{a}\right)^k\right];\ k=2$$

$$\Delta P = \frac{4L\,K}{w}\left[10.8\,\frac{\overline{U}}{w}\right]^n \int_0^{\pi/2} \sin^{2n+1}\theta\, d\theta$$



4. Conclusion

- Good agreement between COMSOL results and analytical expressions of the literature

- The 2D-Helle-Shaw approach is valid when $\varepsilon < ½$

  for rectangular and pillared channels and Newtonian fluids. (It is less accurate for aspect ratios slightly above 1).

- COMSOL assistance has produced a correlation for pressure drop in pillared microchannels (small aspect ratio)

- COMSOL assistance has produced a correlation for pressure drop of viscoelastic 'Ostwald fluids' in square microchannels.

- Pressure drops of visco-elastic fluids in pillared channels are still under investigation

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Entrepreneurship
Team work
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