Presented at the COMSOL Conference 2010 Paris

Design of Acoustic Metamaterials based on the Concept of Dual Transmission Line

***Anne-Sophie Moreau*[1], *Hervé Lissek*[1] *and Frédéric Bongard*[2]

[1]*Ecole Polytechnique Fédérale de Lausanne, Switzerland*

[2]*JAST SA, Switzerland*

Laboratoire d'Electromagnétisme et d'Acoustique

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Layout of the presentation

- Introduction & Objectives
- Transmission line concept
  - Conventional & Dual acoustic TL
- Lumped element models
  - Membrane, Stub & CRLH TL
- Finite element models
  - Membrane, Stub & CRLH TL
- Methodologies for the assessment
  - Lumped & Finite element models
- Results
  - Bloch, Scattering parameters & Unbalanced case
- Conclusion

Introduction

- **Acoustic metamaterials**: artificial structures using inclusions of elements, whose dimensions are smaller than the wavelengths of interest, so as to enact effective macroscopic behavior not readily available in nature

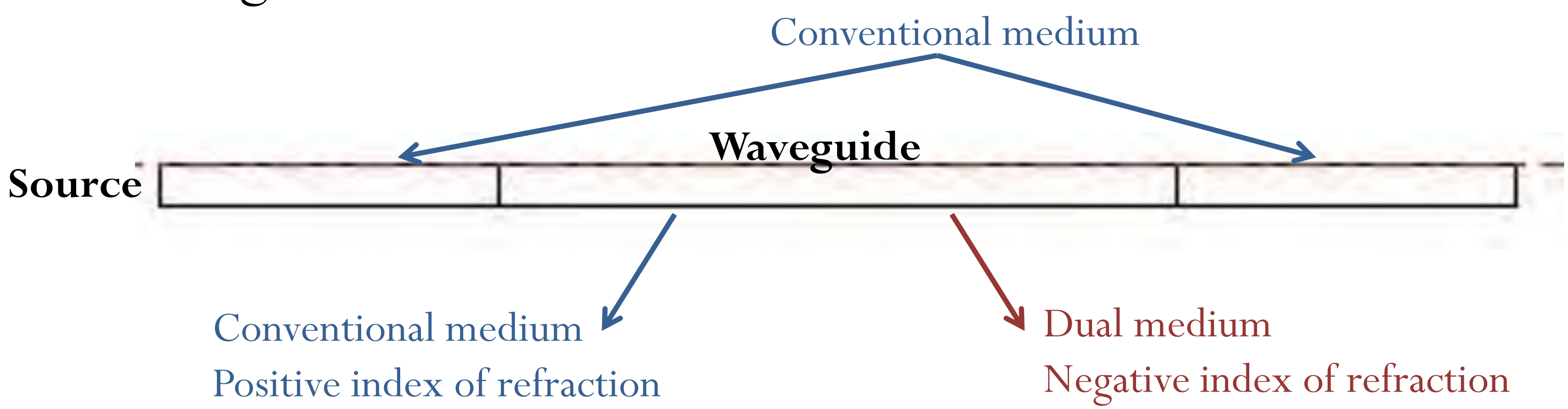
- Growing interest for acoustic metamaterials

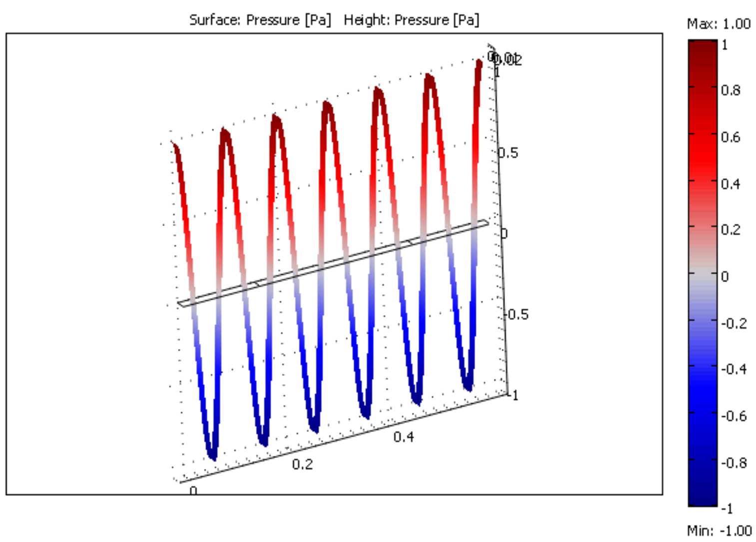
⮡ Capability to achieve new properties like negative refraction

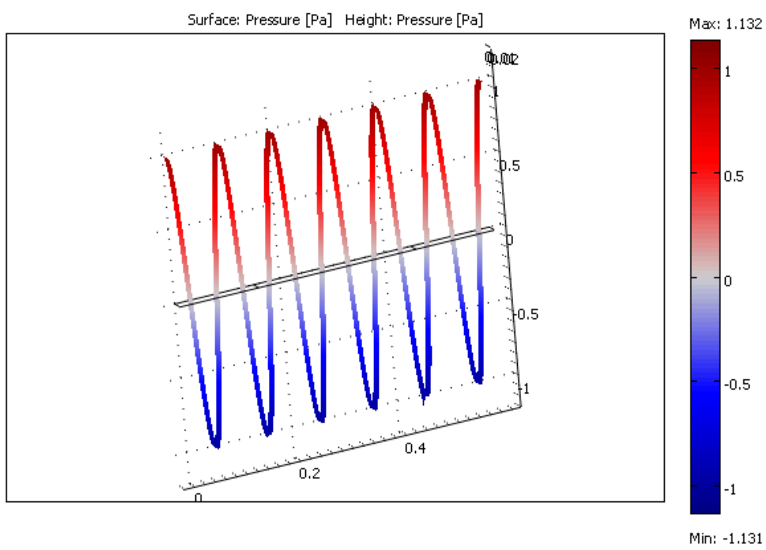
- Lots of development in Electromagnetics
- Proposed structure based on transmission line concept: acoustic waveguide loaded with membranes and open radial stubs

Objectives

- Negative refraction: an illustration

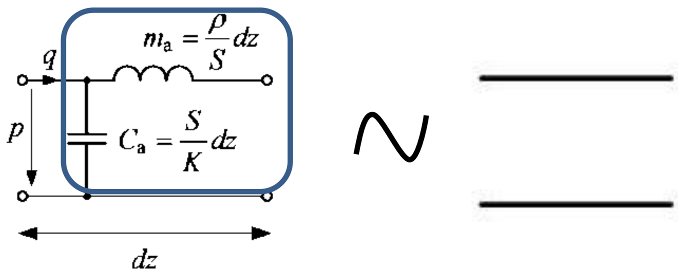




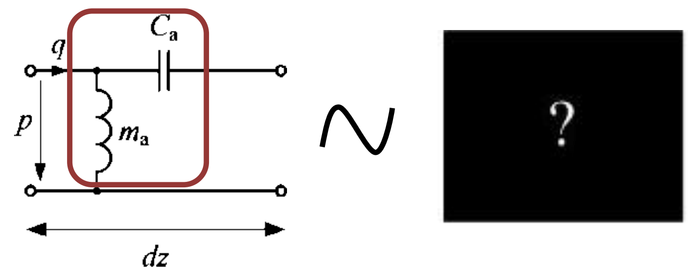


Transmission line concept

- Conventional acoustic TL
  - Describe the propagation of waves
  - Positive index of refraction



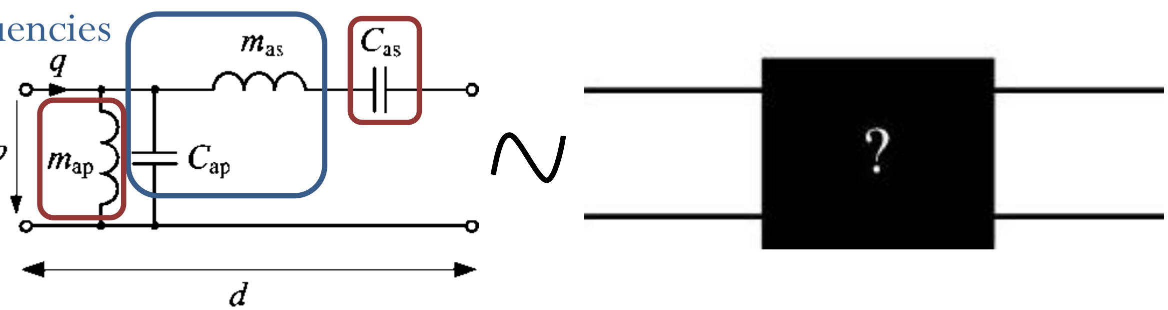
- Dual acoustic TL
  - Dual topology
  - Negative index of refraction



- Unit cell of the composite right/left-handed (CRLH) TL
  - Dual acoustic TL has to load a conventional TL
  - Equivalent circuits assumed lossless

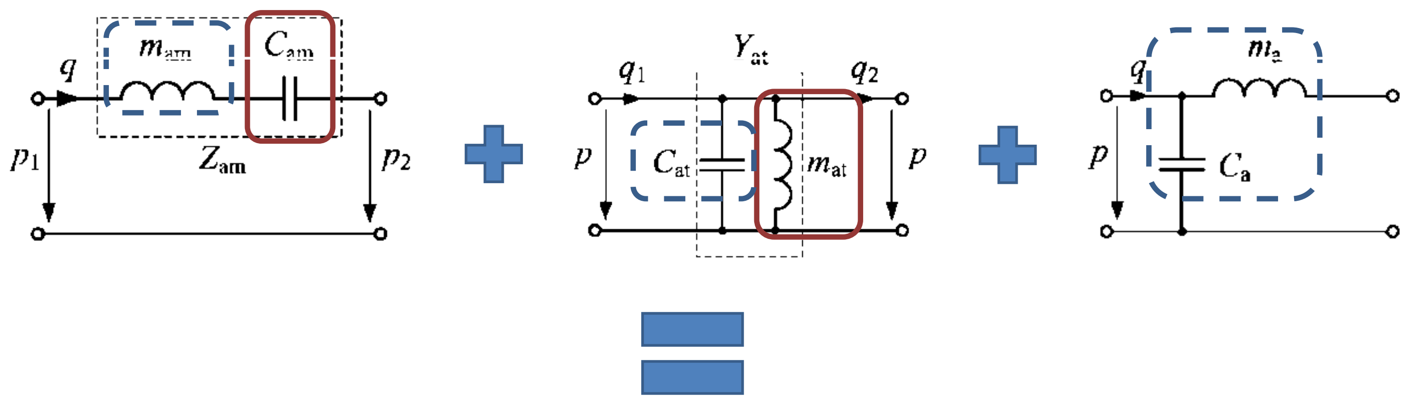
Conventional TL for high frequencies

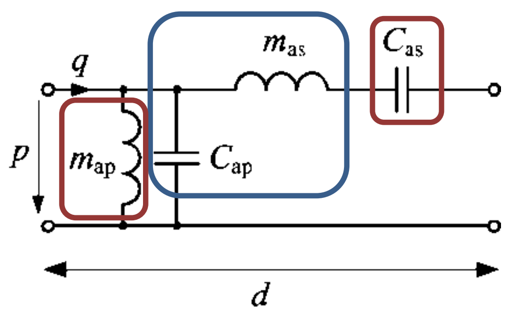
Dual TL for low frequencies



Lumped element models

- Membrane
  - Implements a **series compliance**
  - Mechanical element
- Stub
  - Implements a **parallel mass**
  - Acoustic element
- Host waveguide
  - Conventional TL





$$\begin{cases} m_{as} = m_{am} + m_a \\ C_{ap} = C_{at} + C_a \end{cases} \quad and \quad \begin{cases} m_{ap} = m_{at} \\ C_{as} = C_{am} \end{cases}$$

Lumped element models

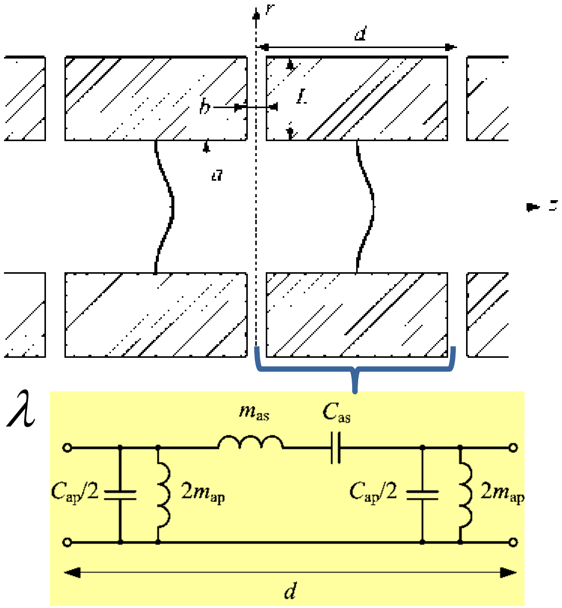
- CRLH TL
  - Balanced condition between RH & LH bands

$$m_{as}C_{as} = m_{ap}C_{ap}$$

  - Balanced condition for f_0 = 1kHz
  - Lattice constant d small compared to the wavelength λ

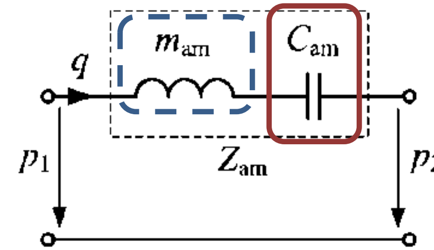
$$d/\lambda = 0.1 \text{ at } f_0 \Longrightarrow d = 34 \text{ mm}$$

  - Dimensions and values of masses & compliances



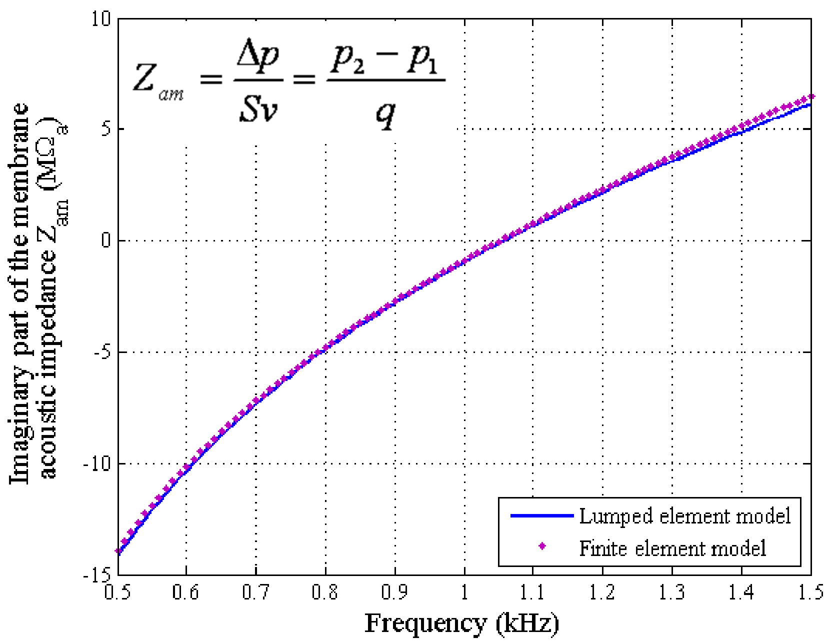
	Dimension	Value
a	Membrane radius	9.06 mm
h	Membrane thickness	125 µm
b	Stub thickness	1 mm
L	Stub length	43.5 mm
d	Lattice constant	34 mm

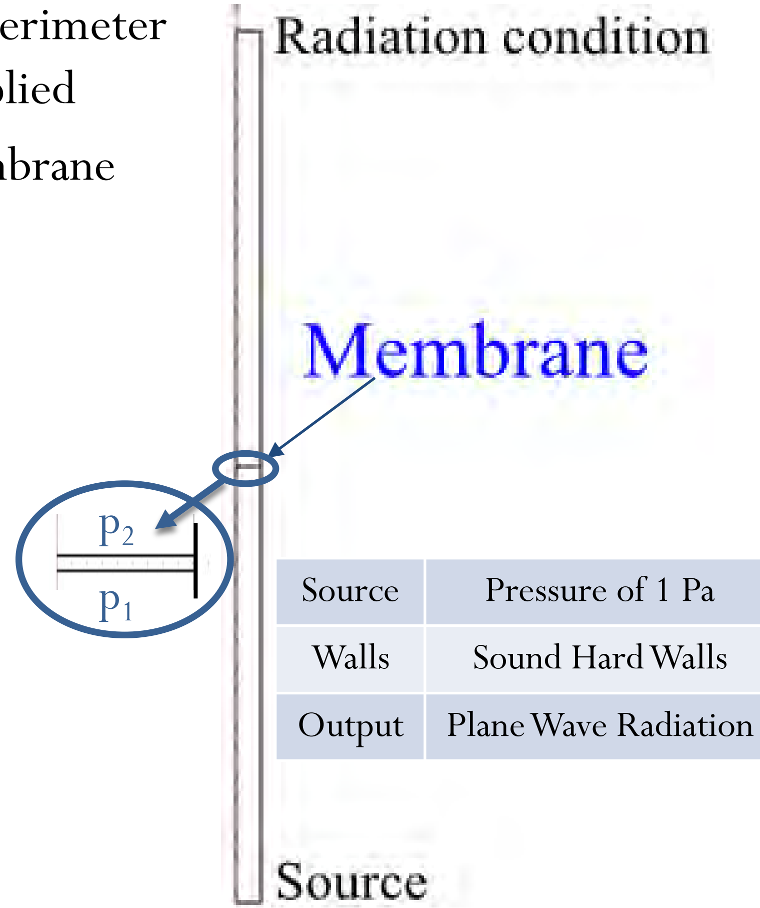
		Value (kg/m[4])		Value (x10[-12] m[3]/Pa)
TL	m_a	156.1	C_a	63.6
Membrane	m_{am}	1296	C_{am}	17.43
Stub	m_{at}	351.1	C_{at}	8.34
Series	m_{as}	**1452.1**	C_{as}	**17.43**
Parallel	m_{ap}	**351.1**	C_{ap}	**71.94**



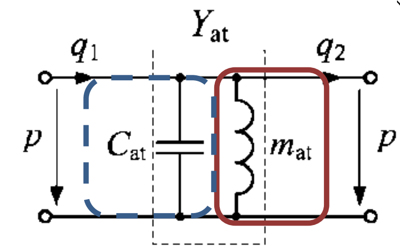
Finite element models

- Membrane
  - 2D axi-symmetric acoustic structure interaction model
  - Thin circular membrane clamped at its perimeter modeled as a thin plate: no tension is applied
  - Δp: average pressure applied on the membrane
  - v: average velocity of the membrane



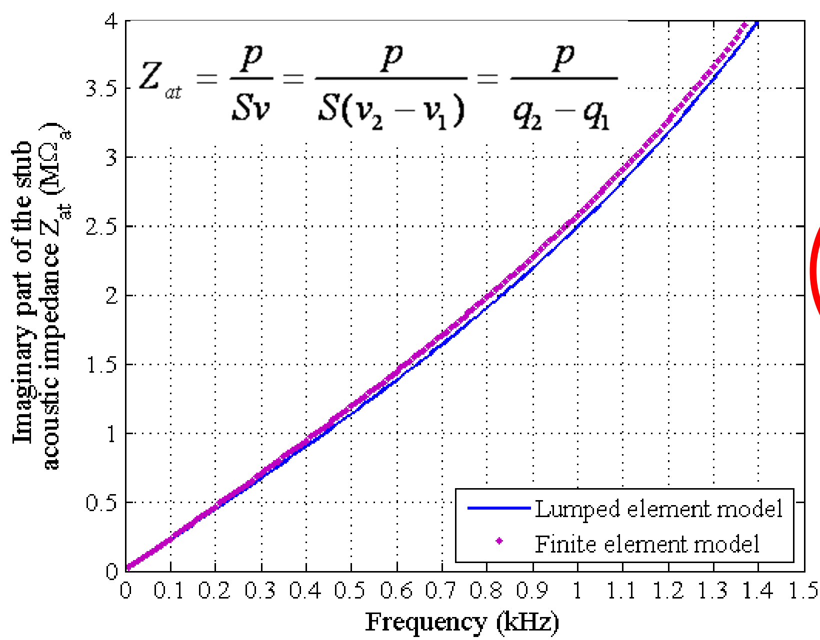


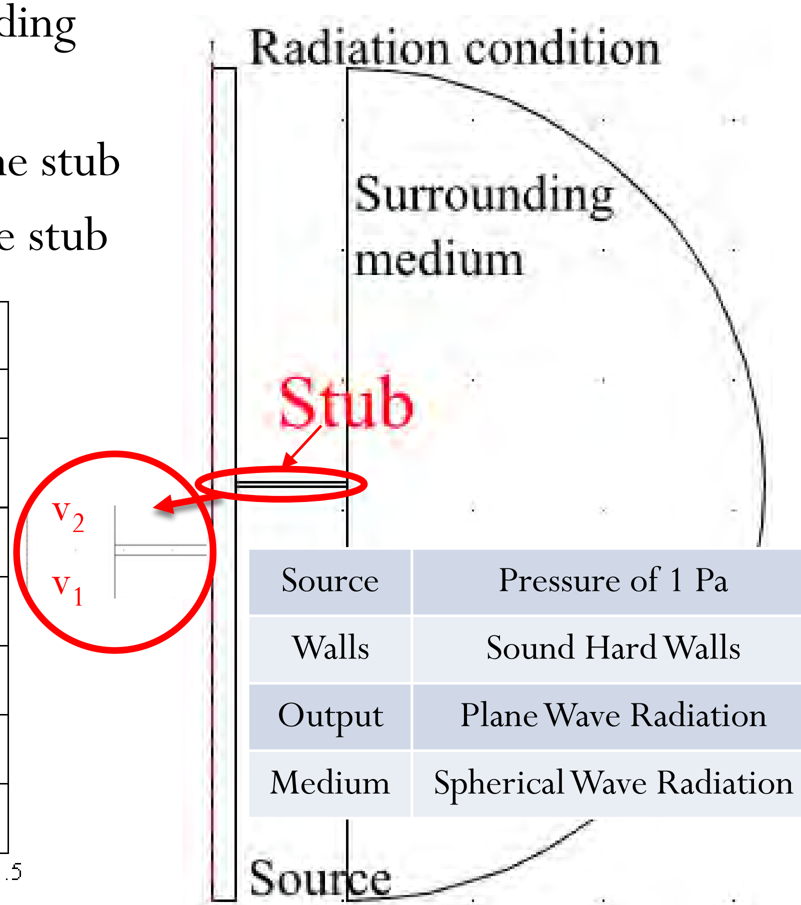
Source	Pressure of 1 Pa
Walls	Sound Hard Walls
Output	Plane Wave Radiation



Finite element models

- Stub
  - 2D axi-symmetric pressure acoustic model
  - Radial open tube radiating in a surrounding medium
  - *p*: average pressure at the entrance of the stub
  - *v*: average velocity at the entrance of the stub

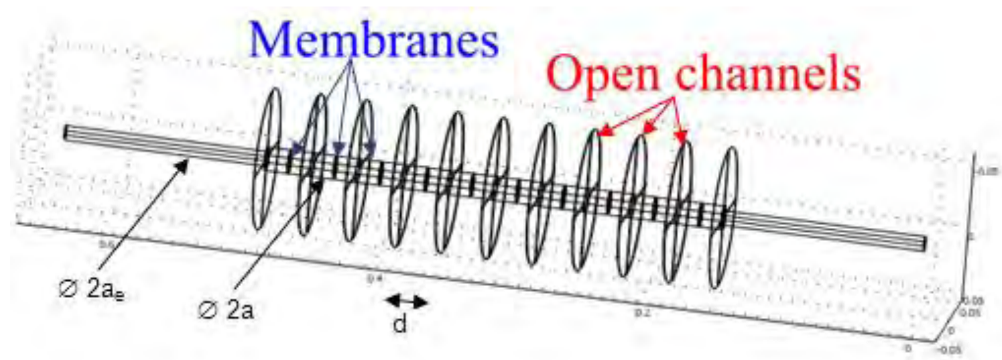




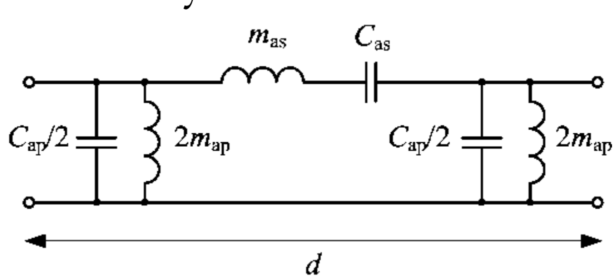
Source	Pressure of 1 Pa
Walls	Sound Hard Walls
Output	Plane Wave Radiation
Medium	Spherical Wave Radiation

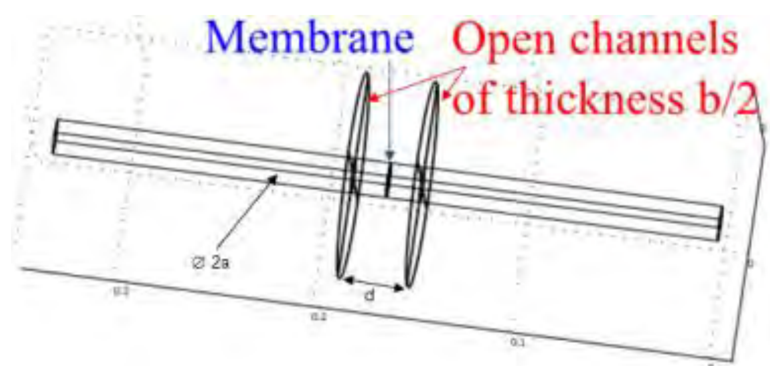
Finite element models

- CRLH-TL
  - 2D axi-symmetric acoustic structure interaction model
  - Periodic structure with the same boundary conditions as before
  - Connection to an adapted host waveguide to avoid reflection at the interfaces
  - CRLH-TL with 10 cells



  - 1 symmetric cell





Methodologies for the assessment

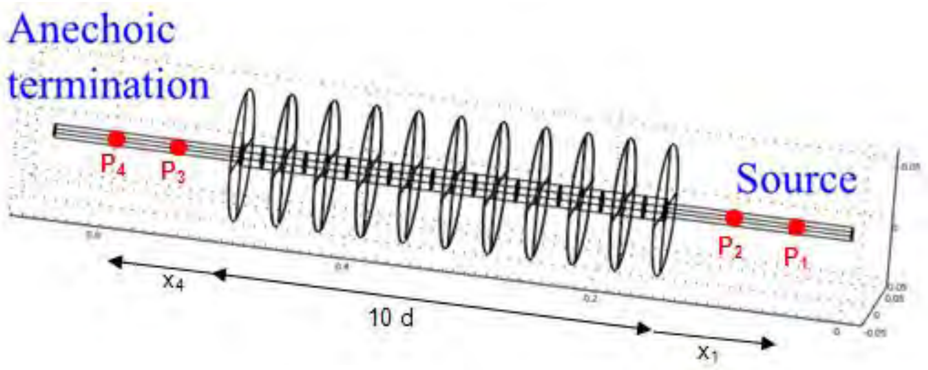
- Lumped element model
  - Computation of the Bloch parameters
  - Description of the propagation of waves in periodic structures
  - Pulsation of the two branches $\omega_R = \frac{1}{\sqrt{m_{as}C_{ap}}}$ *and* $\omega_L = \frac{1}{\sqrt{m_{ap}C_{as}}}$
  - Bloch impedance Z_B and dispersion diagram $\beta_B d$

$$
\Rightarrow \begin{cases} \cos(\gamma_B d) = 1 - \dfrac{\left(\dfrac{\omega}{\omega_R} - \dfrac{\omega_L}{\omega}\right)^2}{2} \\[2ex] Z_{B,\pi} = \dfrac{\sqrt{\dfrac{m_{as}}{C_{ap}}}}{\sqrt{1 - \dfrac{\left(\dfrac{\omega}{\omega_R} - \dfrac{\omega_L}{\omega}\right)^2}{4}}} \end{cases}
$$

Methodologies for the assessment

- Finite element model
  - Four probes sensing pressure

Distance	Value
$x_1 = -x_4$	0.1 m
$x_2 = -x_3$	0.05 m
s	0.05 m



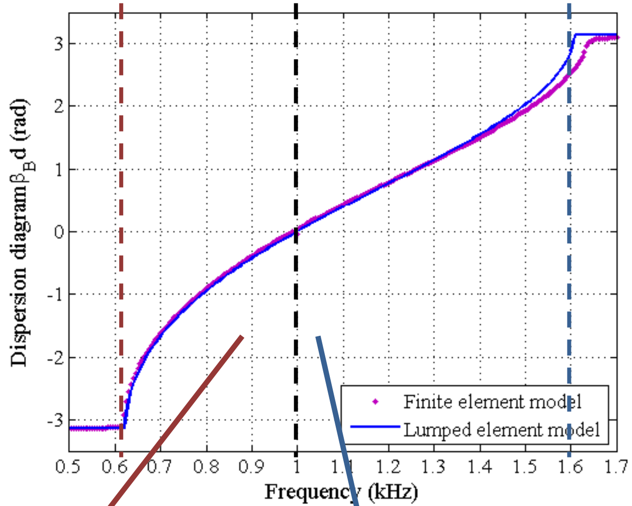
  - Propagation of plane waves under $f < 0.586\frac{c}{2a} = 11kHz$
  - 2 & 4 microphones methods
  - Scattering parameters: 1 & 10 cells
  - Bloch parameters: 1 cell

$$\begin{cases} R = \dfrac{H_{12} - e^{-jks}}{e^{jks} - H_{12}} e^{2jkx_1} \\ T = \dfrac{C}{A} = \dfrac{P_3 e^{jkx_4} - P_4 e^{jkx_3}}{P_1 e^{jkx_2} - P_2 e^{jkx_1}} \end{cases}$$

$$\begin{cases} \cos(\gamma_B d) = \dfrac{1 - R^2 + T^2}{2T} \\ Z_B = \pm Z_0 \sqrt{\dfrac{(1+R)^2 - T^2}{(1-R)^2 - T^2}} \end{cases}$$

Results

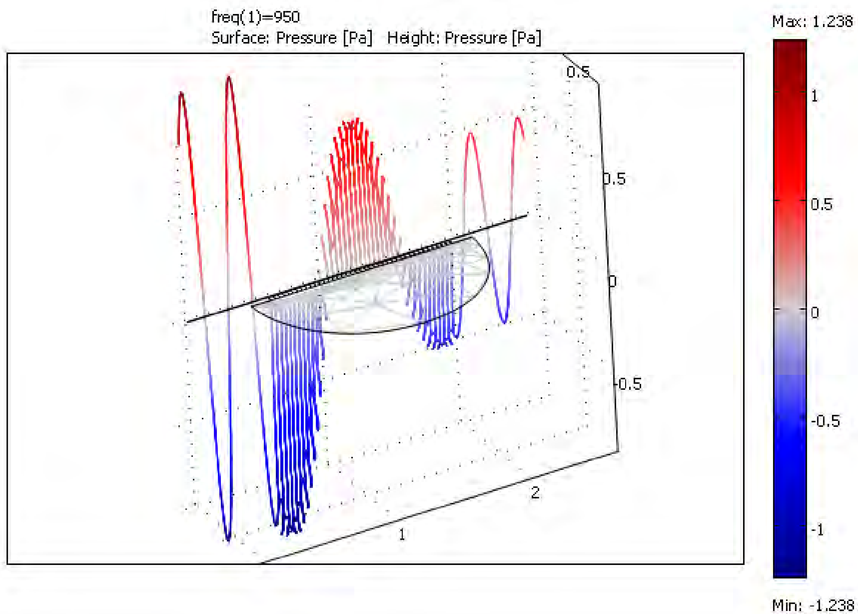
- Bloch parameters



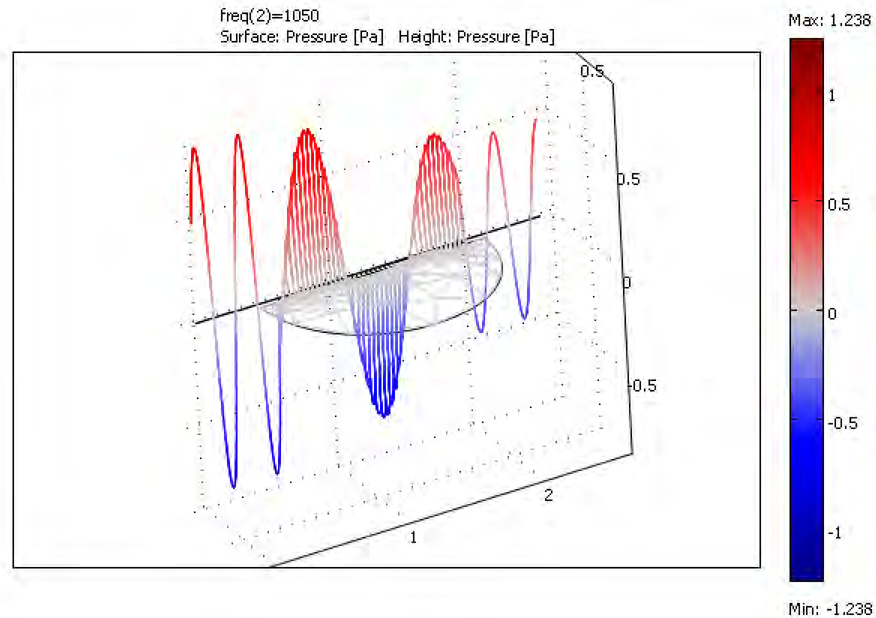
Negative refraction

Positive refraction

Pressure at $f = 950$ Hz

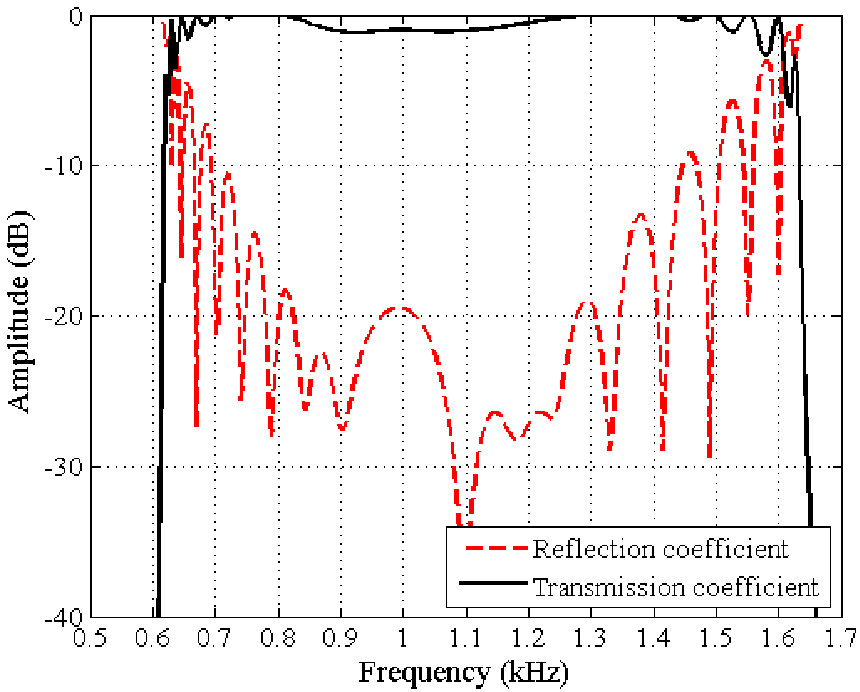


Pressure at $f = 1050$ Hz



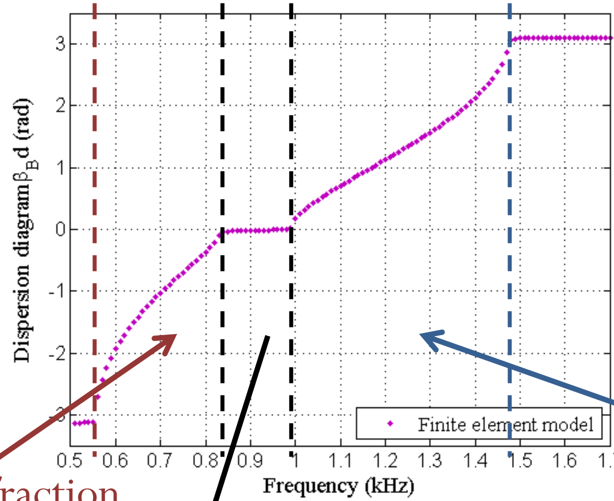
Results

- Scattering parameters
  - 2 & 4 microphones methods can be realized experimentally
  - 2 microphones ⟶ reflection coefficient
  - 4 microphones ⟶ transmission coefficient
  - LEM & FEM match well (previous results) so FEM results can be used as a reference for experimental results



Results

- Unbalanced case
  - Finite element model
  - $L = 80$ mm

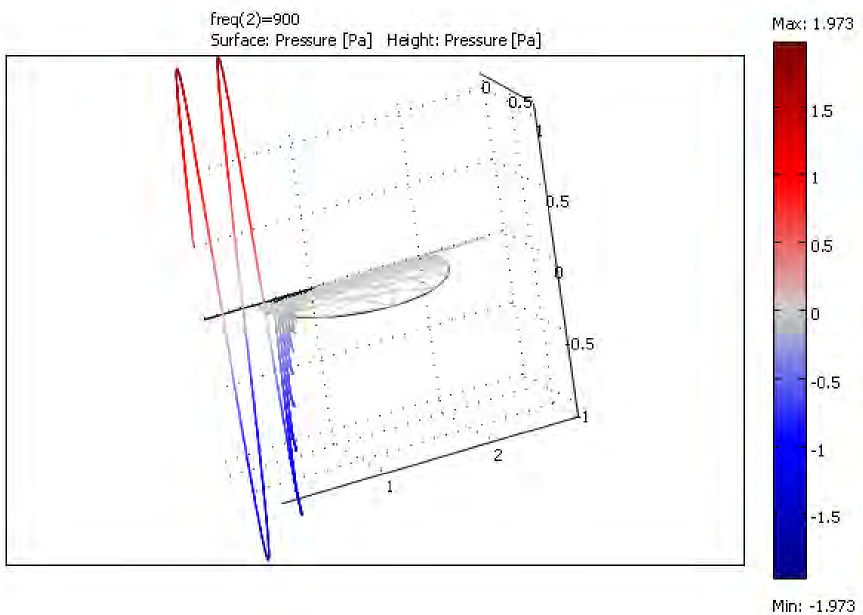


Negative refraction

Positive refraction

No propagation

Pressure at $f = 900$ Hz



Conclusion

- Circuit theory concepts efficiently used to design a TL-based metamaterial
- Inclusion of mechanical elements to realize series compliance
- Proposed structure: negative refractive band of almost one octave with a balanced condition & unbalanced case
- LEM & FEM models confirm performances
- Future experimental measurement of scattering parameters to validate the results
- Further work in FEM to predict performances of a 2D version of this structure

Thank you for your attention

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Corresponding author : herve.lissek@epfl.ch

This work was supported by the Swiss National Scientific Foundation

under research grant 200021-130255