Presented at the 2011 COMSOL Conference

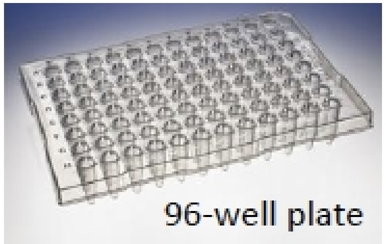
Study of AC electrothermal Fluid Flow Models

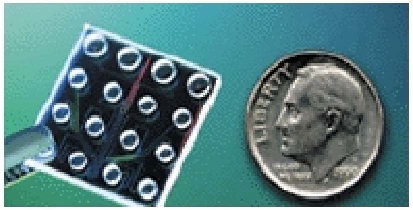
Sophie Loire

Paul Kauffmann, Igor Mezić

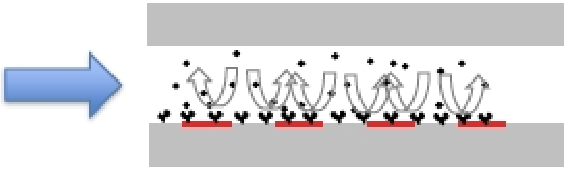
Department of Mechanical Engineering,
University of California, Santa Barbara

Motivation: Microfluidic manipulation





Caliper's LabChip for DNA & protein analysis



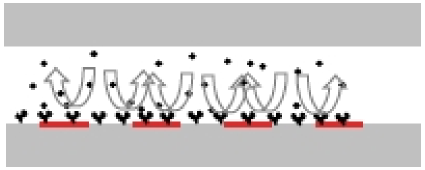
Mixing improves reaction rates

- Fully integrated lab on chip
- Mixing, concentration, pumping, separation of fluids and particles in microchannels
- Example: Mixing for bioassays

  goal: Improve reaction rate in traditional and microarray biosensors via mixing.

Motivation: Microfluidic manipulation

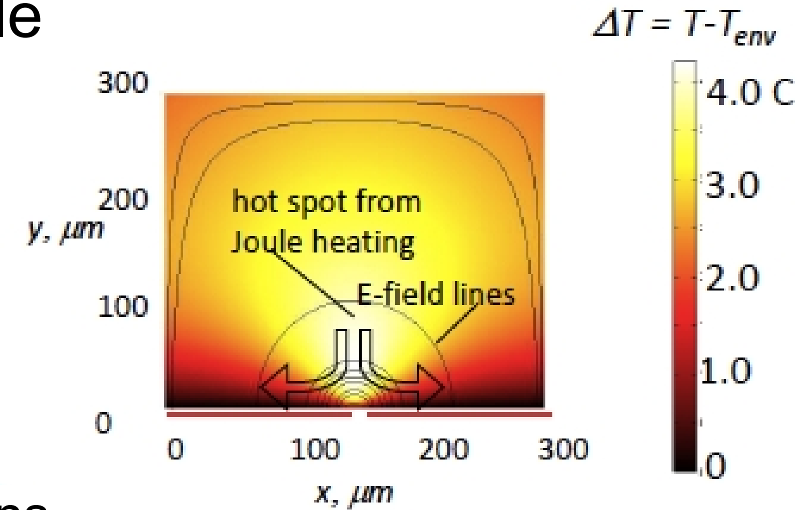
- **Solution:** Integrated electrodes generating AC electrothermal mixing



Mixing improves reaction rates

- **Electrothermal force:**

  interaction of gradients in conductivity and permittivity (produced by Joule Heating) with electric field



- **Advantages:**
  - Easy integration
  - AC=>avoid electrolysis
  - Effective for high conductivity solutions

Outline

- Standard numerical model for ACET
- Experimental discrepancy for high conductivity
- Solution: strong thermo-electric coupling using Comsol software
- Results
- Conclusion and Future works

Standard numerical model for ETF

[1] A. Castellanos, A. Ramos, A. Gonzales, N. G. Green, and H. Morgan, Electrohydrodynamics and dielectrophoresis in microsystems: scaling law, *Journal of Physics D: Applied Physics*, vol. 36, pp. 2584–2597 (2003).

- Electric Field

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1, \text{ with } \mathbf{E}_1 \ll \mathbf{E}_0$$
$$\nabla \mathbf{E}_0 = 0$$

- Temperature Field

$$k_m \nabla^2 T + \frac{\sigma_m}{2} |\mathbf{E}_0|^2 = 0$$

- Electrothermal Body Force

$$< \mathbf{F}_{ET} > \;=\; 0.5\varepsilon_m \left[\left(\frac{1}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) - \frac{1}{\sigma_m} \left(\frac{\partial \sigma_m}{\partial T} \right) \right) \frac{\nabla T \cdot \mathbf{E}_0}{1 + (\omega\tau)^2} \mathbf{E}_0 - \frac{0.5}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) \nabla T |\mathbf{E}_0|^2 \right]$$

- Fluid velocity Field

$$\mu_m \nabla^2 \mathbf{u} + < \mathbf{F}_{\mathbf{ET}} > = \nabla P$$

Standard numerical model for ETF

Estimation of velocity amplitude voltage dependence

$k_m \nabla^2 T + \frac{\sigma_m}{2} |\mathbf{E}_0|^2 = 0$ => Temperature: $\Delta T \sim V^2$

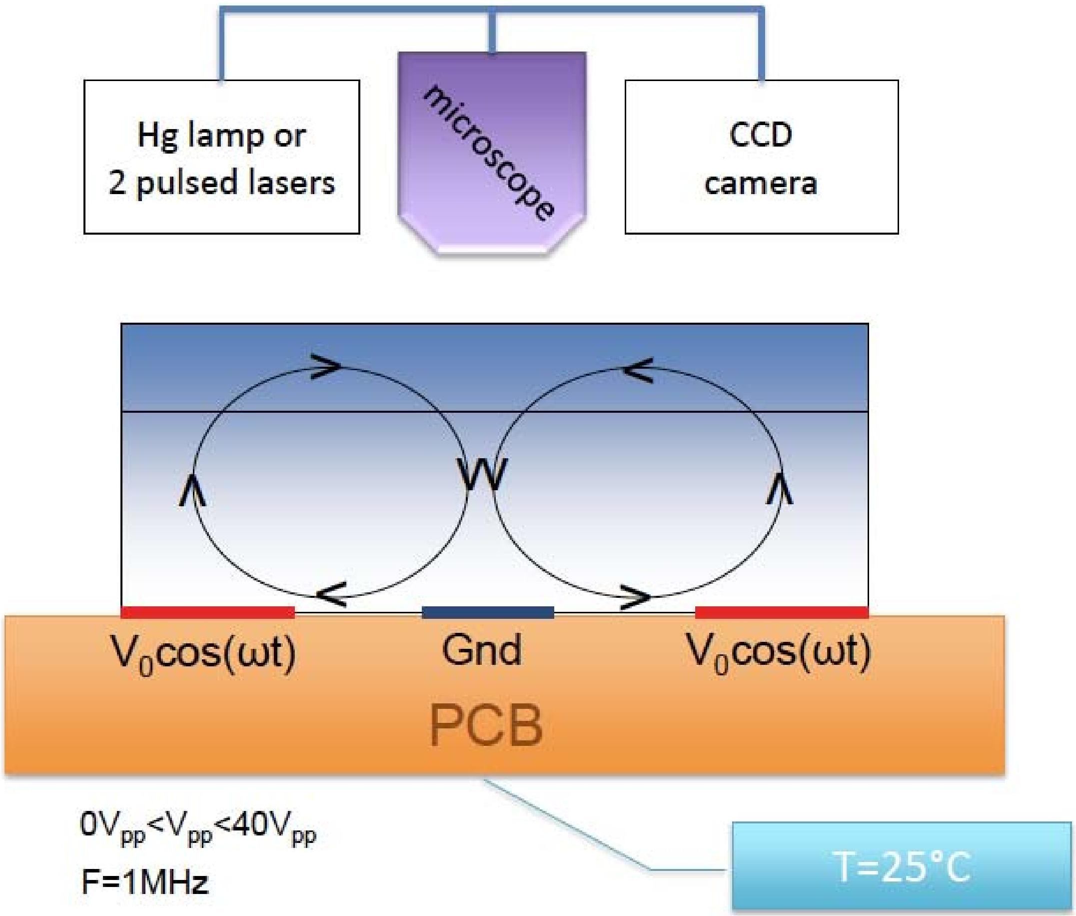
$\mu_m \nabla^2 \mathbf{u} + < \mathbf{F_{ET}} > = \nabla P$ => Velocity: u ~ $< \mathbf{F_{ET}} >$

$$< \mathbf{F}_{ET} > = 0.5\varepsilon_m \left[\left(\frac{1}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) - \frac{1}{\sigma_m} \left(\frac{\partial \sigma_m}{\partial T} \right) \right) \frac{\nabla T \cdot \mathbf{E_0}}{1+(\omega\tau)^2} \mathbf{E_0} - \frac{0.5}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) \nabla T |\mathbf{E_0}|^2 \right]$$

=> u ~ $\Delta T\ E^2$

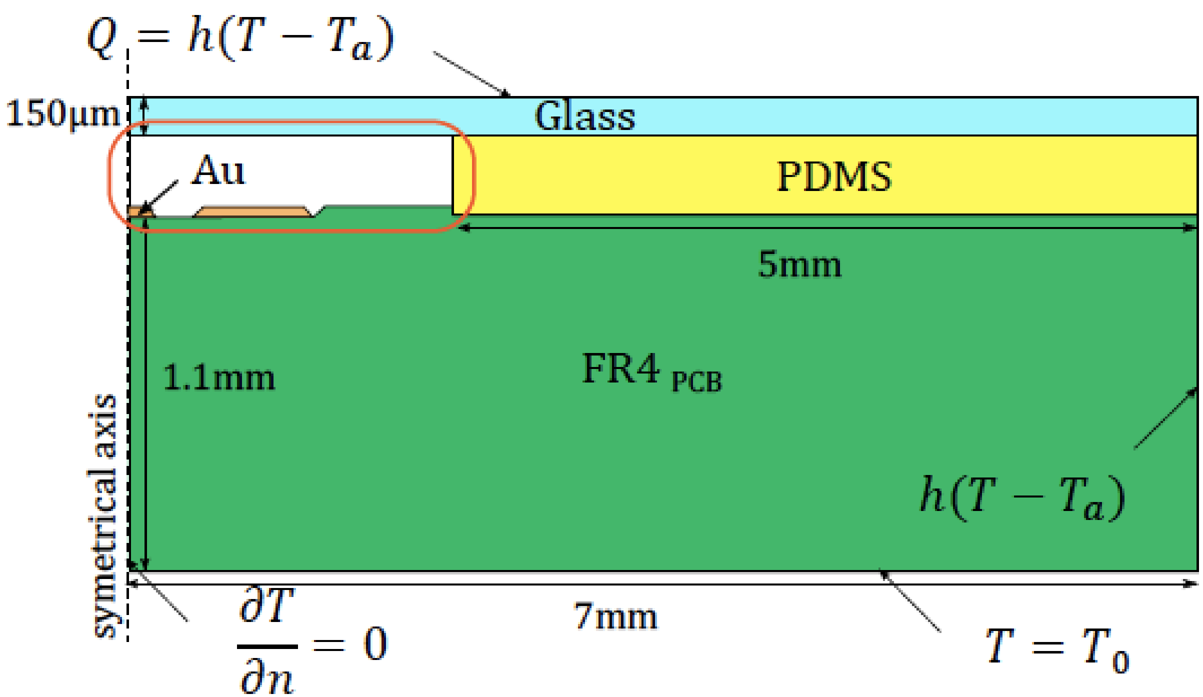
u ~ V^4

Experimental Setup:

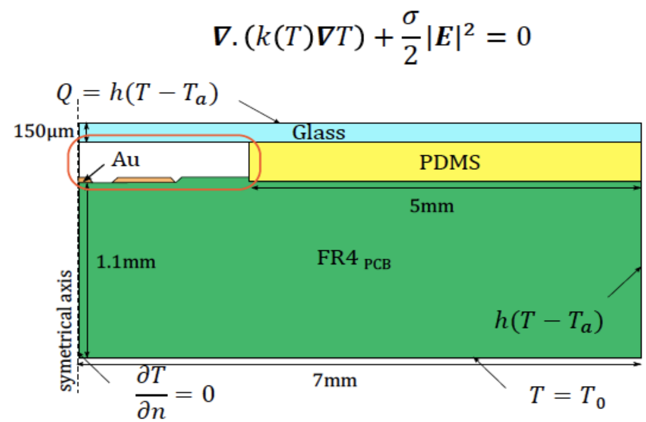


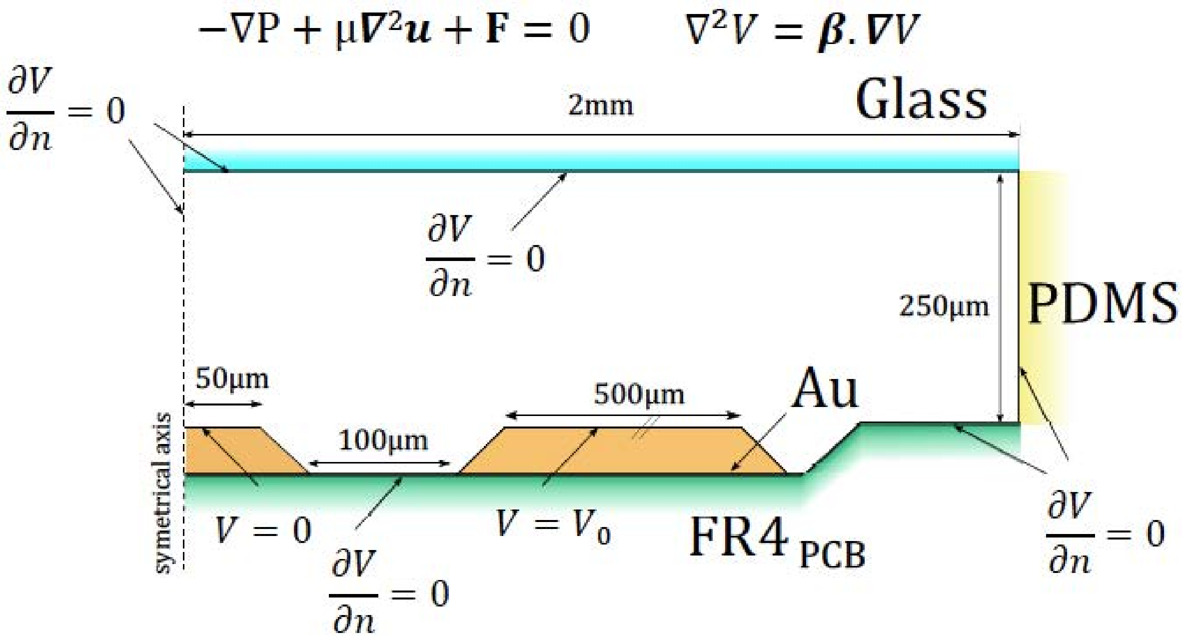
Comsol Implementation

$$\nabla.(k(T)\nabla T) + \frac{\sigma}{2}|E|^2 = 0$$



Comsol Implementation





Standard numerical model for ETF

Estimation of velocity amplitude voltage dependence

$k_m \nabla^2 T + \frac{\sigma_m}{2} |\mathbf{E}_0|^2 = 0$ => Temperature: $\Delta T \sim V^2$

$\mu_m \nabla^2 \mathbf{u} + < \mathbf{F_{ET}} > = \nabla P$ => Velocity: $u \sim < \mathbf{F_{ET}} >$

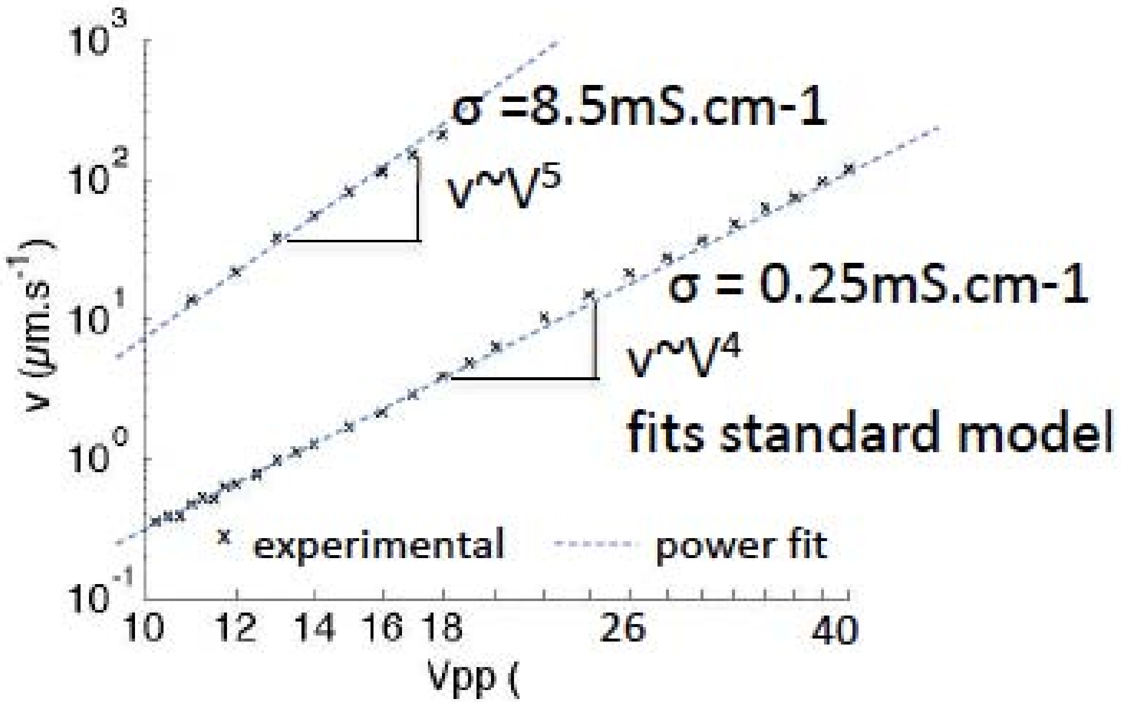
$$< \mathbf{F}_{ET} > = 0.5\varepsilon_m \left[\left(\frac{1}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) - \frac{1}{\sigma_m} \left(\frac{\partial \sigma_m}{\partial T} \right) \right) \frac{\nabla T \cdot \mathbf{E_0}}{1 + (\omega\tau)^2} \mathbf{E_0} - \frac{0.5}{\varepsilon_m} \left(\frac{\partial \varepsilon_m}{\partial T} \right) \nabla T |\mathbf{E_0}|^2 \right]$$

=> $u \sim \Delta T\, E^2$

$u \sim V^4$

Experimental discrepancy

Experimental discrepancy: For high conductivity $v \sim V^5$



Strong Thermo-Electric Coupling

Solution:

1. Remove small perturbed field simplification

2. $\sigma_m=\sigma_m(T)$, $k_m=k_m(T)$, $\mu_m=\mu_m(T)$

Low temperature gradient Standard theory	High temperature gradient Strong thermo-electric coupling

$\omega\tau \ll 1$

Low temperature gradient — Standard theory

$$\boldsymbol{E} = \boldsymbol{E}_0 + \boldsymbol{E}_1, with\ |\boldsymbol{E}_1| \ll |\boldsymbol{E}_0|$$

$$\boldsymbol{E}_0 = -\boldsymbol{\nabla} V$$

$$\begin{cases} \nabla^2 V = \mathbf{0} \\ \boldsymbol{\nabla}.(k_m\, \boldsymbol{\nabla} T) + \dfrac{\sigma_m}{2}|\boldsymbol{E}|^2 = 0 \\ -\boldsymbol{\nabla} P + \boldsymbol{\nabla}.(\mu_m \boldsymbol{\nabla} \boldsymbol{u}) + \boldsymbol{F}_{ET} = \mathbf{0} \end{cases}$$

High temperature gradient — Strong thermo-electric coupling

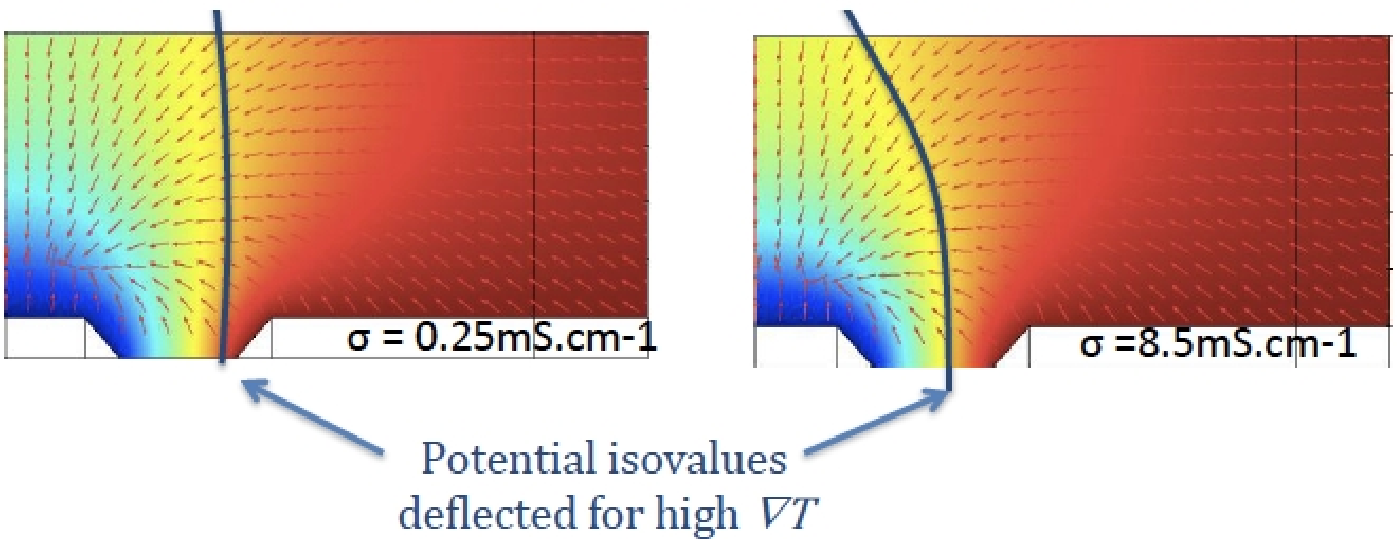
$$\boldsymbol{\nabla} \boldsymbol{E}_R = -\frac{1}{\sigma_m}\frac{\partial \sigma_m}{\partial T}\nabla T.\boldsymbol{E}_R,\quad \boldsymbol{E}_I = 0$$

$$\begin{cases} \nabla^2 V = -\dfrac{1}{\sigma_m}\dfrac{\partial \sigma_m}{\partial T}\nabla T.\nabla V \\ \boldsymbol{\nabla}.(k_m(T)\boldsymbol{\nabla} T) + \dfrac{\sigma_m(T)}{2}|\boldsymbol{E}|^2 = 0 \\ -\boldsymbol{\nabla} P + \boldsymbol{\nabla}.(\mu_m(T)\boldsymbol{\nabla} \boldsymbol{u}) + \boldsymbol{F}_{ET} = \mathbf{0} \end{cases}$$

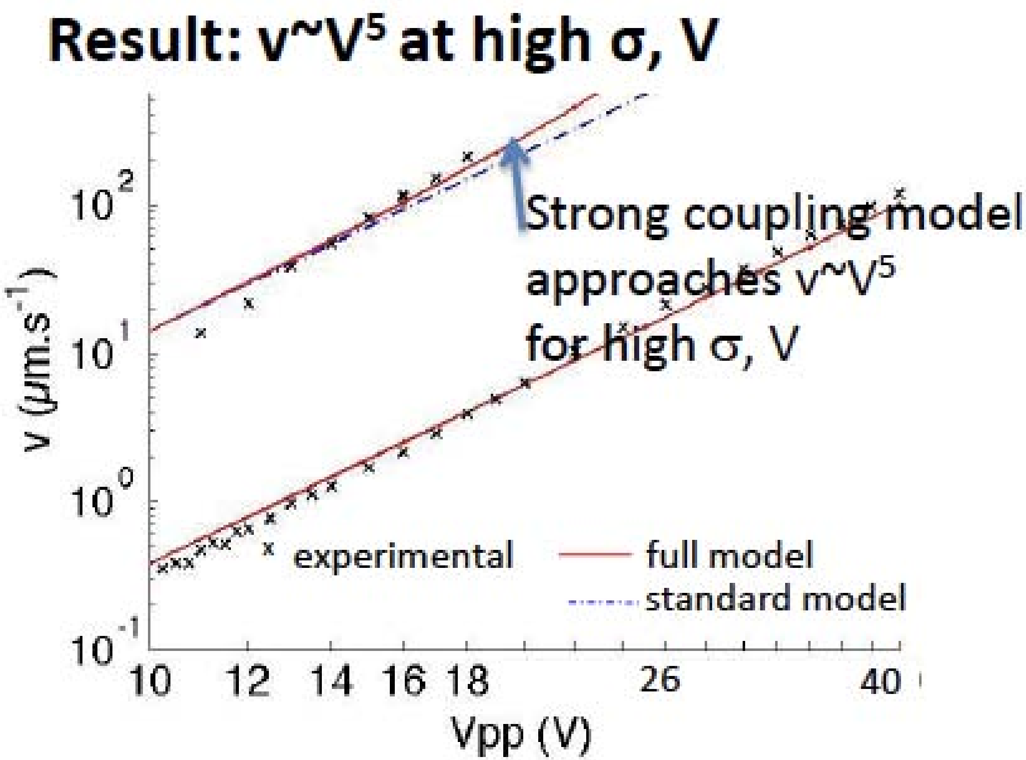
Results:

$$\nabla^2 V = \beta \cdot \nabla V$$

Result: advection of E-field by conductivity gradient



Results:


Result: v~V⁵ at high σ, V
Strong coupling model
approaches v~V⁵
for high σ, V
v (μm.s⁻¹)
10²
10¹
10⁰
10⁻¹
10
12
14
16
18
26
40
Vpp (V)
experimental
full model
standard model

Conclusion

Strong thermo-electric coupling and
Temperature dependent expression of parameters

are necessary to correctly model ACET at high gradient of temperature.

Future Works

- Include buoyancy to the model.
- Check for which parameters buoyancy is NOT negligeable.