# Aquifer Pumping Test Evaluation

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## Introduction

In a pumping test the drawdown of the water table in a piezometer is recorded, as response to pumping from a nearby well (Kruseman & de Ridder 1990). Pumping tests belong into the common toolbox of hydro-geologists, used to obtain basic parameters for aquifer characterization.

There are various approaches to evaluate the recordings from pumping tests. It is common practice to determine transmissivity (T) and storativity (S) from fitting 1D-analytical solutions to the observed drawdown. However, often situations are dealt with in which the simple 1D-approach is not justified. These cases can be handled by numerical methods. We present a model and an app tool, built by the COMSOL *Application Builder* that handles general 2D groundwater flow. Parameter estimation is performed using the *Optimization Module*.

## Methods

The numerical model represents a 2D vertical cross-section of the aquifer. We are using 2D cylinder coordinates $r$ (radial) and $z$ (vertical) with the center line of the pumping well as symmetry axis. The differential equation for hydraulic head $h$ in a vertical cross-section of an aquifer is given by (Yeh & Chang 2013):

$$S_s \frac{\partial h}{\partial t} = K_h \left( \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} \right) + K_v \frac{\partial^2 h}{\partial z^2} \qquad (1)$$

$S_s$ denotes the specific storativity, $K_h$ the horizontal and $K_v$ the vertical hydraulic conductivity. Equation (1) is derived from the fluid mass conservation principle and Darcy's Law for porous media flow. If the variable of hydraulic head $h$ is replaced by drawdown $s=h_0-h$, where $h_0$ denotes the initial state, one obtains:

$$S_s \frac{\partial s}{\partial t} = K_h \left( \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) + K_v \frac{\partial^2 s}{\partial z^2} \qquad (2)$$

(Chang *et al*. 2011). For a homogeneous isotropic aquifer with $K = K_h = K_v$ and constant thickness $H$ equation (2) can be modified to:

$$S \frac{\partial s}{\partial t} = T \left( \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} + \frac{\partial^2 s}{\partial z^2} \right) \qquad (3)$$

where $T = K \cdot H$ denotes the aquifer transmissivity and $S = S_s \cdot H$ the storativity.

Analytical solutions can be derived for the 1D case, i.e. if it is assumed that the vertical velocity components and thus the corresponding gradients in equation (3) can be neglected:

$$S \frac{\partial s}{\partial t} = T \left( \frac{\partial^2 s}{\partial r^2} + \frac{1}{r} \frac{\partial s}{\partial r} \right) \qquad (4)$$

The solution for the boundary condition:

$$s(r,t) = \frac{Q}{4\pi T} W\left( \frac{Sr^2}{4Tt} \right) \qquad (5)$$

describes drawdown in the vicinity of an ideal well in a confined aquifer. $Q$ denotes the pumping rate. Equation (5) is mostly referred to as Theis solution (Theis 1937). It describes drawdown $s$ as a function of the radial distance to the well centre-line $r$ and time $t$ and is expressed by the function $W(u)$, which among hydro-geologists is known as *well-function*. In mathematics it is referred to as exponential integral, defined by:

$$W(u) = E_1(u) = \int_u^\infty \frac{\exp(-\varsigma)}{\varsigma} d\varsigma \qquad (6)$$

The variable $u = Sr^2 / 4Tt$ combines both space and time variables with the parameters storativity $S$ and the transmissivity $T$.

The condition of vanishing vertical velocity components is probably not fulfilled in non-ideal wells, where the well screen does not extend across the entire thickness of the aquifer. The derivation also requires that the aquifer has a large extend. Furthermore Theis solution may also not be used for unconfined aquifers, and if there is aquifer recharge or leakage from other sources. These conditions can be considered in a 2D model.

## Model Set-up

Using COMSOL Multiphysics a 2D model was set-up for a vertical cross-section, which represents the aquifer from the borehole wall to the reach of the well. The following geometric properties can be considered: the thickness of the aquifer, the location of the well screen and the depth position of the observation point. Some of these parameters are often known, but the common evaluation practice doesn't take advantage of this knowledge.

In COMSOL one has two options to model the groundwater equations. One may use equations (1), (2) or (3) directly and solve for hydraulic head *h*. The alternative is to use the Darcy mode for porous media flow. Then the pressure $p = \rho g z$ (with fluid density ρ, acceleration due to gravity $g$ and depth below the water table $z$) becomes the dependent variable instead of the hydraulic head.

The model region is a rectangle with aquifer thickness as height *H* and the reach of the well as length *L*, as shown in Figure 1. Screen position, aquifer thickness, horizontal extension and the exact position of the observation points are further geometric entities taken into account in the 2D approach.

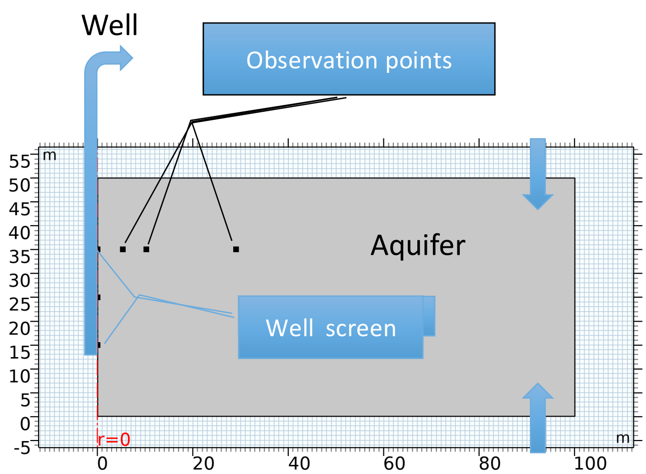


**Figure 1**. Concept of 2D model

## Parameter Estimation

In the model parameter estimation is performed using the COMSOL *Optimization Module*. While $T$ and *S* parameters are determined by default, other unknown parameters can be included in the estimation procedure, such as the reach of the well, the thickness of the aquifer, the initial position of the water table, groundwater recharge and the ratio of hydraulic conductivities.

The optimization module contains three options for the iterative optimization method: BOBYQA, Levenberg-Marquard and SNOPT. The latter two are gradient-based while BOBYQA (bound optimization by quadratic approximation) is a gradient free method. BOBYA and SNOPT offer the option to set lower and upper bounds for the parameter range, which is very convenient in practical applications. For all methods the user can specify an optimality tolerance. The number of iterations is restricted by a maximum number of model evaluations that can also be specified by the modeler.

## Pumping Test App

In order to enable the use of the model in practical applications by users with no modeling skills, an app was created, using the COMSOL Multiphysics Application Builder. Figure 2 depicts the Pump Test App, including the results of an application case.

The input parameters and their values and units are listed at the top left of the panel: reach, well radius, aquifer thickness, initial head, initial values for transmissivity and storativity, ratio of conductivities $K_h / K_v$, groundwater recharge, pumping rate, top and bottom of screen, as well as the observation point coordinates in relation to the well center and the initial head. The user may change all values for these input parameters.

Below the input parameter one finds the button to browse the file that contains the input data recorded during pumping. The data have to be given in two columns, one containing the times of measurements, the other observed drawdown. One may use text files, tabulator separated values (tsv-) or comma separated values (csv-) files. The user has to specify the time and length units of the data on the file.

On the panel there are some options to control the optimization method: the maximum number of model evaluations, the tolerance and the least-squares method option. The latter concerns the computation of the objective function that is to be minimized.

The 'Compute' button initiates the model run. A progress bar appears at the bottom of the panel, showing the progress of the optimization procedure. There is also a button to stop the model run.

During the model run current results are shown in the list on the right of the 'Compute' button. They show the iteration numbers and the corresponding objective function.

After the optimization is finished final results are shown in the in the main graph. It depicts drawdown measurements (red) and computed drawdowns (blue) for the optimized $(T,S)$-parameter pair. The grey plot depicts the drawdown according to the analytical solution (5) for the estimated parameter set. One has to expect big deviations between the 1D and 2D solutions in case of significant flow components in vertical direction, as shown in the example depicted in the figure.

The optimized parameter values can be taken as last values from the list below the main graph. The plot on the lower right visualizes the advancement of the objective function during the optimization process. The row index is the iteration number.

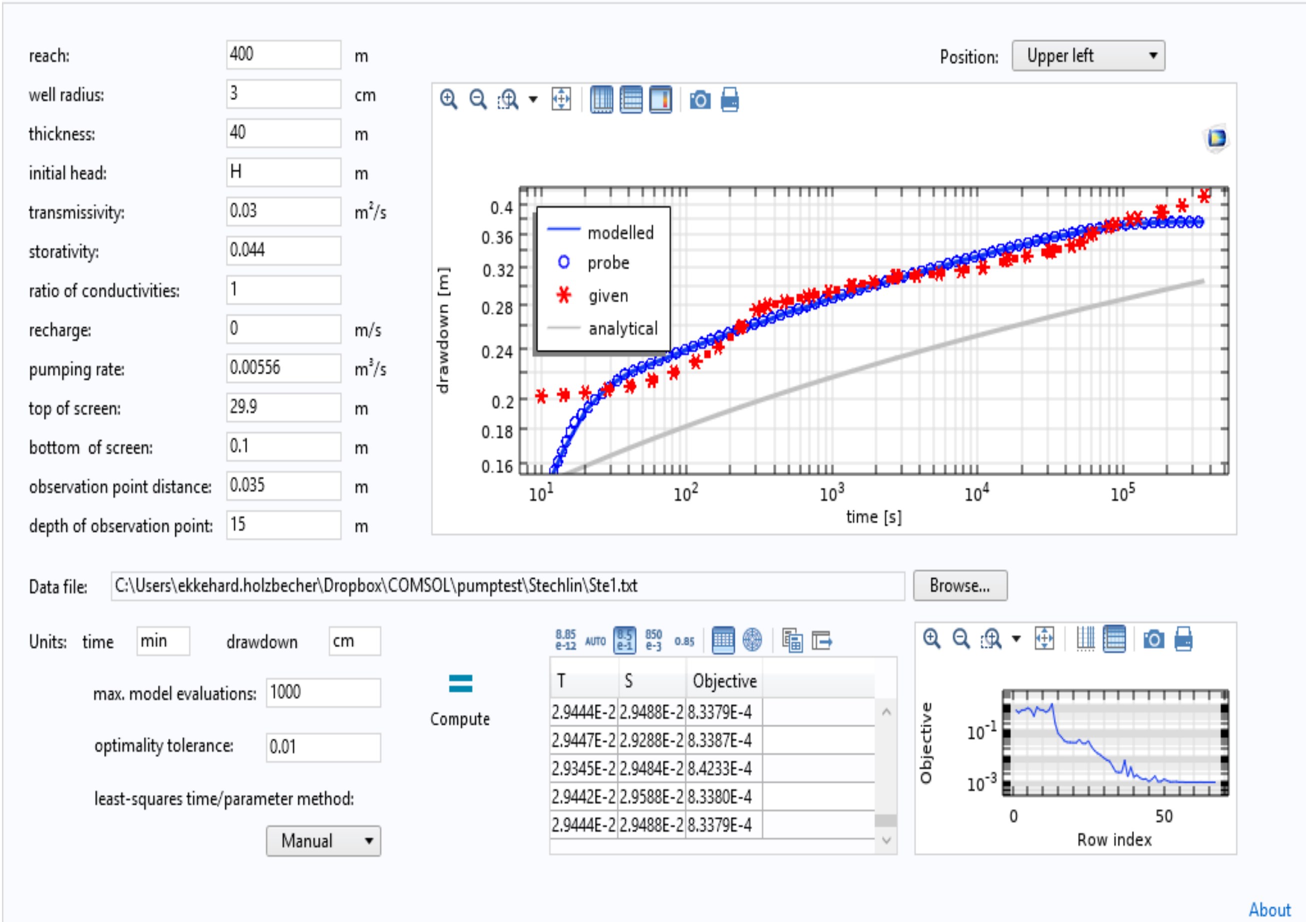


**Figure 2.** The Pump Test App with example evaluation

## Application

The model was applied for several pumping tests that are reported in the literature. Here we present results for the Oude Korendijk pumping test that was performed near to Rotterdam in 1963 (de Wit, 1963). The case is treated in several publications (Samani *et al*. 2007, Barua & Bora 2010) and textbooks (Kruseman & de Ridder 1990).

A shallow aquifer of 10 m thickness was pumped for 14 hours at an average pumping rate of 788 $m^3$/d or 32.8 $mm^3$/h. The drawdown was monitored in two observation boreholes, in distances of 30 and 90 m from the pumping well.

The well screen extends over the entire thickness of the aquifer. The observation points are assumed to be located in medium depth between the top and bottom of the aquifer.

Here the Levenberg-Marquard method was used for the parameter estimation. The optimality tolerance was set to 0.001, and the maximum of number of evaluations to 1000. The manual least-squares method was chosen for computing the objective function.

The drawdown for the observation point in 30 m distance is plotted in a double-logarithmic coordinate system in Figure 3. Red markers indicate the measured data. The blue plot represents the drawdown according to the 2D model with the best fit ($T$,$S$)-values. The grey plot shows the drawdown according to the 1D analytical solution using the very same parameter values.

The results of the 2D model depend on the geometrical model parameters. The highest sensitivity is due to the reach, the horizontal extension of the model. Table 1 gives an impression, how much the parameters change due to changes in $L$.

**Table 1:** Estimated parameter values using the 2D model

| Reach [m] | Transmissivity [$m^2$/d] | Storativity S•$10^4$[-] | Objective function |
|---|---|---|---|
| 400 | 473.4 | 1.93 | 0.049 |
| 500 | 505.8 | 1.68 | 0.032 |
| 1000 | 603.0 | 1.07 | 0.011 |

Lower objective functions are obviously obtained for larger horizontal extensions of the model region - as far as the latter were increased. Thus one may favor the values given in the last row, if nothing is known about the reach of the well. Including the reach in the estimation procedure led to an optimum at $L$ =1150 m.

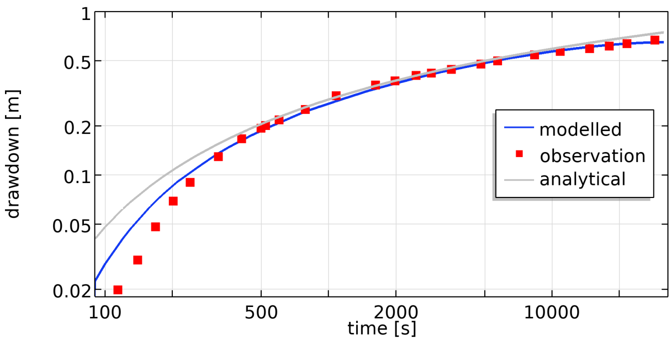


**Figure 3**. Oude Korendijk pump test evaluation, observation well in 30 m distance

The result from classical evaluation based on the 1D analytical solution is $T$ = 480.5 $m^2/d$ and $S$= 1.125 $10^{-4}$. Obviously it makes a difference, taking the limited reach of the well into account.

## Conclusions

2D numerical methods can be used successfully for the evaluation of pumping tests. They are superior to methods based on 1D analytical solutions, as they are built on less restrictive assumptions. Moreover, one can take advantage of a better site characterization by including known parameters that are not taken into account by classical evaluation methods.

The described model was verified using data performed for tests in confined aquifers, and compared to results obtained the common 1D approach. In most applications it was possible to improve the fit in comparison to the 1D evaluation.

An application was created using the COMSOL Application Builder. The app allows users with limited or without modeling knowledge to apply the advanced 2D-based pumping test evaluation methods.

Currently the app deals with confined aquifers only. However, 2D models for unconfined aquifers can be set up similarly (Holzbecher 2019) and, using the Application Builder, can be transformed into an application as outlined above.

## References


1. Chang Y.-C., Chen G.-Y., Yeh H.-D.,Transient flow into a partially penetrating well during the constant-head test in unconfined aquifers, *J. of Hydraul. Eng.*, **137**, 1054-1063 (2011)
2. Barua G., Bora S.N., Hydraulics of a partially penetrating well with skin zone in a confined aquifer, *Adv. in Water Resources*, **33**, 1575-1587 (2010)
3. Holzbecher E., Pumping Tests: Comparison of 2D Numerical and 1D Analytical Solutions, Europ. COMSOL Conf., Proceedings, Cambridge (2019)
4. Kruseman G.P., de Ridder N.A., Analysis and Evaluation of Pumping Test Data, Intern. Inst. for Land Reclamation and Improvement (ILRI), The Netherlands (1990)
5. Samani N., Gohari-Moghadam M., Safavi A.A., A simple neural network model for the determination of aquifer parameters, *Journal of Hydrology*, **340**, 1-11 (2007)
6. Theis C.V., The relation between the lowering of the piezometric surface and the rate and duration of discharge of a well using groundwater storage, *Trans. Am. Geophys. Union*, **16**, 519-524 (1937)
7. Wit K.E., De hydrologische bodemconstanten in de polder ‚De Oude Korendijk'…, Inst. voor Culturtechniek en Waterhuishouding, Wageningen, No. 190 (1963) (in Dutch)
8. Yeh H.-D., Chang Y.-C., Recent advances in modeling of well hydraulics, *Adv. in Water Res.*, **51**, 27-51 (2013)